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# Senior School Certificate Examination

## March — 2015

## Marking Scheme — Mathematics 65/1/MT, 65/2/MT, 65/3/MT

***General Instructions :***

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/MT

**EXPECTED ANSWERS/VALUE POINTS**

**SECTION - A**

Marks

1. $\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$ or any other correct example ½ + ½ m

2. Order : 2, degree : 2, Product : 4 ½ + ½ m

3. $\frac{dy}{dx} = -\alpha A \sin \alpha x + \alpha B \cos \alpha x$ ½ m

$$\left.\begin{aligned} \frac{d^2y}{dx^2} &= -\alpha^2 (A \cos \alpha x + B \sin \alpha x) \\ \frac{d^2y}{dx^2} &+ \alpha^2 y = 0 \end{aligned}\right\}$$ ½ m

4. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ ½ m

Projection $= \frac{5}{\sqrt{2}}$ ½ m

5. Value $= 3$ 1 m

6. Writing dr's correctly ½ m

D.C'S $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$ ½ m

## SECTION - B

7. $$\begin{array}{l} \\ \text{Family A} \\ \text{Family B} \\ \text{Family C} \end{array} \begin{array}{c} \text{M W C} \\ \begin{pmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{pmatrix} \end{array} \begin{array}{c} \text{Expenses} \\ \begin{pmatrix} 200 \\ 150 \\ 200 \end{pmatrix} \end{array} = \begin{array}{c} \text{Family expenses} \\ \begin{pmatrix} 1050 \\ 1150 \\ 2300 \end{pmatrix} \end{array}$$ 2 m

Expenses for family A = ₹ 1050
Expenses for family B = ₹ 1150
Expenses for family C = ₹ 2300 1 m

Any relevant impact 1 m

8. $\tan^{-1}x + \tan^{-1} y = \frac{\pi}{2} - \tan^{-1} z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \cot^{-1}z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}\left(\frac{1}{z}\right)$ as $z > 0$ 1 m

$\frac{x+y}{1-xy} = \frac{1}{z}$ ½ m

$xy + yz + zx = 1$ ½ m

9. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$R_1 \to R_1 + R_2 + R_3$

$(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ 1 m

$C_1 \rightarrow C_1 - C_2,\ C_2 \rightarrow C_2 - C_3$

$$(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix} = 0$$ 2 m

$(a + b + c)\ (ab + bc + ca - a^2 - b^2 - c^2) = 0$

given $a \neq b \neq c$, so $ab + bc + ca - a^2 - b^2 - c^2 \neq 0$ ½ m

$\Rightarrow\ (a+b+c) = 0$ ½ m

10. Let $x = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1 m

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$

$$\begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$ 1½ m

$a + 4b = -7,\ c + 4d = 2,\ 2a + 5b = -8,\ 2c + 5d = 4$

Solving $a = 1,\ b = -2,\ c = 2,\ d = 0$ 1 m

$\therefore\ x = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ ½ m

OR

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$|A| = 1 \neq 0$, $A^{-1}$ will exist ½ m

$$\text{adj } A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$ (Any four correct Cofactors : 1 mark) 2 m

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$ ½ m

$$A^{-1} A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ 1 m

11. $f(x) = |x-3| + |x-4|$

$$= \begin{cases} 7-2x, & x < 3 \\ 1, & 3 \le x < 4 \\ 2x-7, & x \ge 4 \end{cases}$$ 1 m

L. H. D at x = 3 $\lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$

$$\lim_{x \to 3^-} \frac{6-2x}{x-3} = -2$$

R. H. D at x = 3 $\lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$

$$= \frac{1-1}{x-3} = 0$$

L. H. D ≠ R. H. D $\therefore$ $f(x)$ is not diffrentiable at x = 3 1½ m

L. H. D at x = 4 $\lim_{x \to 4^-} \frac{f(x) - f(4)}{x - 4}$

$= \frac{1-1}{x-4} = 0$

R. H. D at x = 4 $\lim_{x \to 4^+} \frac{f(x) - f(4)}{x - 4}$

$\lim_{x \to 4^+} \frac{2x - 7 - 1}{x - 4} = 2$

L. H. D at x = 4 $\neq$ R.H.D at x = 4

f(x) is not differentiable at x = 4 1½ m

12. $y = x^{e^{-x^2}}$

$\log y = e^{-x^2} \log x$ 1 m

Diff. w. r. t x

$\frac{1}{y} \frac{dy}{dx} = \frac{e^{-x^2}}{x} + \log x \, e^{-x^2} (-2x)$ 2 m

$\frac{dy}{dx} = y\left(\frac{e^{-x^2}}{x} - 2x \log x \, e^{-x^2}\right)$ ½ m

$= x^{e^{-x^2}} e^{-x^2} \left(\frac{1}{x} - 2x \log x\right)$ ½ m

OR

$\log\sqrt{x^2 + y^2} = \tan^{-1} \frac{x}{y}$

Diff. w. r. t. x

$$\frac{1}{2\left(x^2+y^2\right)}\left(2x+2y\frac{dy}{dx}\right) = \frac{1}{1+\frac{x^2}{y^2}}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right) \qquad 2\text{ m}$$

$$\frac{x+y\frac{dy}{dx}}{x^2+y^2} = \frac{y^2}{x^2+y^2}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right) \qquad 1\text{ m}$$

$$\frac{dy}{dx}\left(y+x\right) = y-x \qquad \tfrac{1}{2}\text{ m}$$

$$\frac{dy}{dx} = \frac{y-x}{y+x} \qquad \tfrac{1}{2}\text{ m}$$

13. $y = \sqrt{x+1} - \sqrt{x-1}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x-1}} \qquad 1\text{ m}$$

$$= \frac{\sqrt{x-1}-\sqrt{x+1}}{2\sqrt{x^2-1}} \qquad \tfrac{1}{2}\text{ m}$$

$$4\left(x^2-1\right)\left(\frac{dy}{dx}\right)^2 = y^2 \qquad \tfrac{1}{2}\text{ m}$$

$$4\left(x^2-1\right)2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + 8x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx} \qquad 1\text{ m}$$

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{y}{4} \qquad \tfrac{1}{2}\text{ m}$$

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{y}{4} = 0 \qquad \tfrac{1}{2}\text{ m}$$

14. $\int \frac{1-\cos x}{\cos x\,(1+\cos x)}\ dx$

$= \int \frac{1+\cos x-2\cos x}{\cos x\,(1+\cos x)}\ dx$ 1½ m

$\int \frac{dx}{\cos x} - 2\int \frac{dx}{1+\cos x}$ ½ m

$\int \sec x\, dx - \int \sec^2 \frac{x}{2}\ dx$ 1 m

$\log\left|\sec x+\tan x\right| - 2\tan\frac{x}{2}+c$ 1 m

15. $\int x\sin^{-1}x\ dx$

$\frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\ dx$ 1 m

$\frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}\ dx$ ½ m

$\Rightarrow \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int\sqrt{1-x^2}\ dx - \frac{1}{2}\int \frac{dx}{\sqrt{1-x^2}}\ dx$ 1 m

$\frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right) - \frac{1}{2}\sin^{-1}x+c$ 1½

or $\frac{x^2}{2}\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1}x+c$

16. $\int_0^2 \left(x^2 + e^{2x+1}\right) dx$

$h = \frac{2}{n}$ ½ m

$\int_0^2 \left(x^2 + e^{2x+1}\right) dx = \lim_{h \to 0} h \left[f(0) + f(0+h) + f(0+2h) + \ldots\ldots\right.$

$\left. + \ldots\ldots\ldots f(0+n-1)h \right]$ 1 m

$= \lim_{h \to 0} h \left[h^2 \left(1^2 + 2^2 + \ldots\ldots + (n-1)^2\right)\right.$

$\left. + e\left(1 + e^{2h} + e^{4h} + \ldots\ldots e^{2(n-1)h}\right)\right]$ 1 m

$= \lim_{h \to 0} \frac{(nh)(nh-h)(2nh-h)}{6}$ ½ m

$+ \lim_{h \to 0} e.h.\left(\frac{e^{2nh} - 1}{e^{2h} - 1}\right)$ ½ m

$= \frac{8}{3} + \frac{\left(e^4 - 1\right)e}{2} = \frac{8}{3} + \frac{e^5 - e}{2}$ ½ m

OR

$\int_0^{\pi} \frac{x \tan x \, dx}{\sec x \operatorname{cosec} x}$

$\int_0^{\pi} x \sin^2 x \, dx$ 1 m

Let $I = \int_0^{\pi} x \sin^2 x \, dx$

$$= \int_0^{\pi} (\pi - x) \sin^2 (\pi - x) \, dx$$ ½ m

$$= \int_0^{\pi} (\pi - x) \sin^2 x \, dx$$ ½ m

$$2I = \pi \int_0^{\pi} \sin^2 x \, dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} \, dx$$ ½ m

$$= \frac{\pi}{2} \left[ x - \frac{\sin 2x}{2} \right]_0^{\pi}$$ 1 m

$$= \frac{\pi^2}{2}$$

$$I = \frac{\pi^2}{4}$$ ½ m

17. $\dfrac{x-1}{3} = \dfrac{y-1}{-1} = \dfrac{z+1}{0} = \lambda$ ½ m

$\dfrac{x-4}{2} = \dfrac{y}{0} = \dfrac{z+1}{3} = \mu$ ½ m

$x = 3\lambda + 1, \; y = -\lambda + 1, \; z = -1$ 1 m

$x = 2\mu + 4, \; y = 0, \; z = 3\mu - 1$

At the point of intersection

$\lambda = 1, \; \mu = 0$ 1 m

so $3\lambda + 1 = 4 = 2\mu + 4$ ½

Hence the lines are intersecting

Point of intersection is (4, 0, –1) ½ m

18. Coordinats of Q are $-3\mu+1,\ \mu-1,\ 5\mu+2$ ½ m

D.R's of $\overrightarrow{PQ}$ $-3\mu-2,\ \mu-3,\ 5\mu-4$ 1 m

as $\overrightarrow{PQ}$ is parallel to the plane $x-4y+3z=1$

$1(-3\mu-2)-4(\mu-3)+3(5\mu-4)=0$ 1½ m

$\mu=\frac{1}{4}$ 1 m

OR

The D.R's of the line are 2, –6, 4 1 m

mid point of the line 2, 1, –1 1 m

The plane passes through (2, 1, –1) and is perpendicular to the

plane

eqn. : $2(x-2)-6(y-1)+4(z+1)=0$

$x-3y+2z+3=0$ 1 m

Vector from : $\vec{r}\cdot\left(\hat{i}-3\hat{j}+2\hat{k}\right)+3=0$ 1 m

19. No's divisible by 6 ..................... = 16 1m

No's divisible by 8 ..................... = 12 1m

No's not divisible by 24 .............. = 20 1m

Required probabilty $=\frac{20}{100}=\frac{1}{5}$ 1 m

## SECTION - C

20. For every $a \in A$ , $(a, a) \in R$

$\because$ $|a - a| = 0$ is divisible by 2

$\therefore$ R is reflexive 1 m

For all $a, b \in A$

$(a, b) \in R \Rightarrow |a - b|$ is divisible by 2

$\Rightarrow |b - a|$ is divisible by 2

$\therefore$ $(b, a) \in R$ $\therefore$ R is symmetric 1 m

For all $a, b, c \in A$

$(a, b) \in R \Rightarrow |a - b|$ is divisible by 2

$(b, c) \in R \Rightarrow |b - c|$ is divisible by 2

So, $a - b = \pm 2k$ 1 m

$$\frac{b - c = \pm 2\ell}{a - c = \pm 2m}$$

$\Rightarrow$ $|a - c|$ is divisible by 2

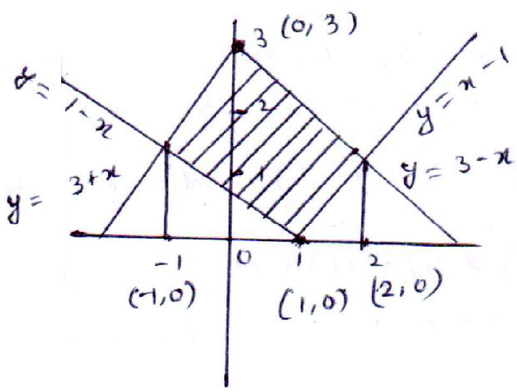
$\Rightarrow$ $(a, c) \in R$

$\Rightarrow$ R is transitive 1 m

Showing elements of $\{1, 3, 5\}$ and 1 m

$\{2, 4\}$ are related to each other

and $\{1, 3, 5\}$ and $\{2, 4\}$ are not related to each other 1 m

21. 


Graph 2 + 2 m

Area of shaded reigon

$$= \int_{-1}^{0} (3+x+x-1)\,dx + \int_{0}^{2} (3-x)\,dx - 2\int_{1}^{2} (x-1)\,dx \qquad 1\text{ m}$$

$$= 2\left.\frac{(x+1)^2}{2}\right]_{-1}^{0} - \left.\frac{(3-x)^2}{2}\right]_{0}^{2} - 2\left.\frac{(x-1)^2}{2}\right]_{1}^{2}$$

$$= 1 - \frac{1}{2}(1-9) - 1 = 4 \text{ sq. units} \qquad 1\text{ m}$$

22. $y = \dfrac{x}{1+x^2}$

$$\frac{dy}{dx} = \frac{1-x^2}{(1+x^2)^2} \qquad 2\text{ m}$$

$$\text{Let } f(x) = \frac{1-x^2}{(1+x^2)^2}$$

$$f'(x) = 0 \Rightarrow \frac{-2x(3-x^2)}{(1+x^2)^3} = 0$$

For max or min $x(3-x^2) = 0 \Rightarrow x = 0$ or $x = \pm\sqrt{3}$ 2 m

$$\left.\begin{aligned} &\text{Calculating } \frac{d^2f(x)}{dx^2} \text{ at } x = 0 < 0 \\ &\qquad\qquad\qquad \text{at } x = \pm\sqrt{3} > 0 \end{aligned}\right\} \qquad 1\text{ m}$$

$$\left.\begin{aligned} &\Rightarrow x = 0 \text{ is the point of local maxima} \\ &\Rightarrow \text{the required pt is } (0, 0) \end{aligned}\right\} \qquad 1\text{ m}$$

23. $\dfrac{dy}{dx} = \dfrac{y^2}{xy - x^2}$

Let $y = vx, \; \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ ½ m

$v + x\dfrac{dv}{dx} = \dfrac{v^2}{v-1}$ 1½ m

$x\dfrac{dv}{dx} = \dfrac{v}{v-1}$

$\dfrac{dx}{x} = \left(\dfrac{v-1}{v}\right) dv$ 1½ m

$\int \dfrac{dx}{x} = \int \left(1 - \dfrac{1}{v}\right) dv$

$\log x = v - \log v + c$ 1 m

$\log y = \dfrac{y}{x} + c \;\text{ or }\; x \log y - y = c\,x$ 1½ m

OR

$\sin 2x \dfrac{dy}{dx} - y = \tan x$

$\dfrac{dy}{dx} - \dfrac{y}{\sin 2x} = \dfrac{\tan x}{\sin 2x}$ 1 m

$\dfrac{dy}{dx} - y(\operatorname{cosec} 2x) = \dfrac{\sec^2 x}{2}$

$P = -\operatorname{cosec} 2x, \; Q = \dfrac{1}{2}\sec^2 x$

$\int P\,dx = -\int \operatorname{cosec} 2x\,dx$

$= -\dfrac{1}{2}\log|\tan x|$

So $e^{\int P\,dx} = \frac{1}{\sqrt{\tan x}}$ 1½ m

Solution is

$$\frac{y}{\sqrt{\tan x}} = \frac{1}{2}\int \frac{\sec^2 x\,dx}{\sqrt{\tan x}} \quad \left( \begin{array}{c} \sqrt{\tan x} = t \\ \Rightarrow \frac{1}{2}\frac{\sec^2 x\,dx}{\sqrt{\tan x}} = dt \end{array} \right)$$ 1½ m

$$\frac{y}{\sqrt{\tan x}} = \sqrt{\tan x} + c$$ 1 m

Getting $c = 1$ ½ m

$\Rightarrow y = \tan x - \sqrt{\tan x}$ ½ m

24. Eqn. of plane

$(x+y+z-6)+\lambda(2x+3y+4z+5) = 0$ 2 m

it passes through $(1,1,1)$

$-3+14\lambda = 0 \Rightarrow \lambda = \frac{3}{14}$ 2 m

Eqn. of plane will be

$20x+23y+26z-69=0$ 1 m

vector from : $\vec{r}\cdot\left(20\hat{i}+23\hat{j}+26\hat{k}\right) = 69$ 1 m

25. Let $E_1$ be the event of following course of meditation and yoga and $E_2$ be the event of following course of drugs 1 m

$P(E_1) = \frac{1}{2}, \; P(E_2) = \frac{1}{2}$ 1 m

$P(A|E_1) = \frac{70 \times 40}{100 \times 100} \quad P(A|E_2) = \frac{75}{100} \times \frac{40}{100}$ 1 m

Formula 1 m

$$\left.\begin{aligned} P(E_1|A) &= \frac{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100}\right)}{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100} + \frac{1}{2} \times \frac{75}{100}\right)} \\ &= \frac{70}{145} = \frac{14}{29} \end{aligned}\right\}$$ 2 m

26. Let the no. of items in the item $A = x$

Let the no. of items in the item $B = y$

(Maximize) $z = 500x + 150y$ 1 m

$x + y \leq 60$

$2500x + 500y \leq 50{,}000$ Graph 2 m

$x, y \geq 0$

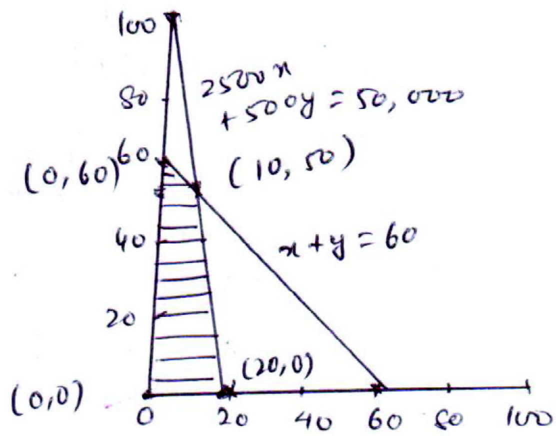
$z(0,0) = 0$

$z(10,50) = 12{,}500$

$z(20,0) = 10{,}000$ 2 m

$z(0,60) = 9{,}000$

Max. Profit $=$ Rs. 12,500 1 m



OR

Let the no. of packets of food $X = x$

Let the no. of packets of food $Y = y$

(minimize) $P = (6x + 3y)$ 1 m

subject to

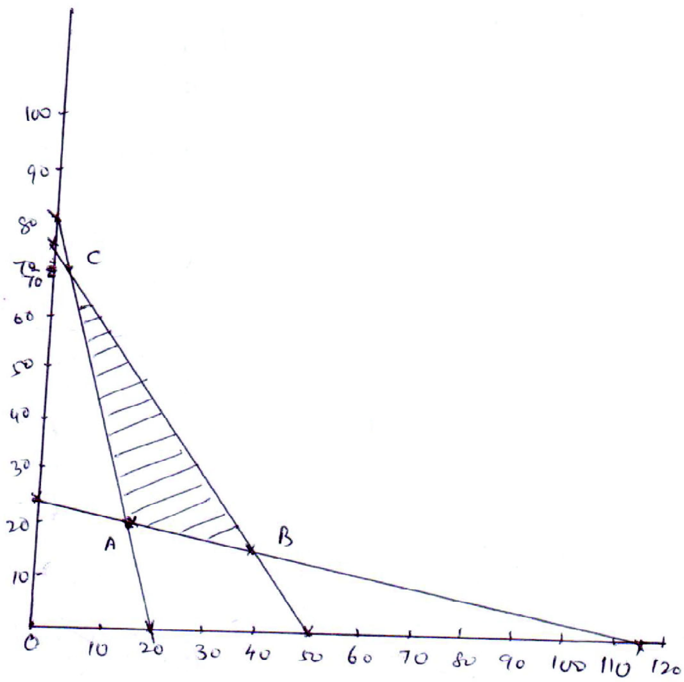
$12x + 3y \geq 240$

$4x + 20y \geq 460$

$6x + 4y \leq 300, \; x, y \geq 0$

or

$$\left.\begin{array}{l} 4x + y \geq 80 \\ x + 5y \geq 115 \\ 3x + 2y \leq 150 \\ x, y \geq 0 \end{array}\right\} \quad 2\text{ m}$$




Correct points of feasible region

A (15, 20), B (40, 15), C (2, 72)

So P (15, 20) = 150

P (40, 15) = 285

P (2, 72) = 228

Graph 2 m

minimum amount of vitamin A = 150 units when 15 packets of food X and 20 packets of food Y are used 1 m

QUESTION PAPER CODE 65/2/MT

**EXPECTED ANSWERS/VALUE POINTS**

**SECTION - A**

Marks

1. Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ ½ m

Projection $= \dfrac{5}{\sqrt{2}}$ ½ m

2. Value $= 3$ 1 m

3. Writing dr's correctly ½ m

D.C'S $\dfrac{3}{13}, \dfrac{4}{13}, \dfrac{12}{13}$ ½ m

4. $\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$ or any other correct example ½ + ½ m

5. Order : 2, degree : 2, Product : 4 ½ + ½ m

6. $\dfrac{dy}{dx} = -\alpha A \sin \alpha x + \alpha B \cos \alpha x$ ½ m

$$\left.\begin{aligned} \frac{d^2y}{dx^2} &= -\alpha^2 (A \cos \alpha x + B \sin \alpha x) \\ \frac{d^2y}{dx^2} + \alpha^2 y &= 0 \end{aligned}\right\}$$ ½ m

## SECTION - B

7. Let $x = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1 m

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$

$$\begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$ 1½ m

$a + 4b = -7,\ c + 4d = 2,\ 2a + 5b = -8,\ 2c + 5d = 4$

Solving $a = 1,\ b = -2,\ c = 2,\ d = 0$ 1 m

$\therefore\ x = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ ½ m

OR

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$|A| = 1 \neq 0,\ A^{-1}$ will exist ½ m

$\text{adj } A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ (Any four correct Cofactors : 1 mark) 2 m

$A^{-1} = \dfrac{\text{adj } A}{|A|} = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ ½ m

$$A^{-1}\ A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ 1 m

8. $f(x) = |x-3| + |x-4|$

$$= \begin{cases} 7-2x, & x<3 \\ 1, & 3\le x<4 \\ 2x-7, & x\ge 4 \end{cases}$$ 1 m

L. H. D at $x = 3$ $\lim_{x\to 3^-} \dfrac{f(x)-f(3)}{x-3}$

$\lim_{x\to 3^-} \dfrac{6-2x}{x-3} = -\ 2$

R. H. D at $x = 3$ $\lim_{x\to 3^+} \dfrac{f(x)-f(3)}{x-3}$

$= \dfrac{1-1}{x-3} = 0$

L. H. D $\neq$ R. H. D $\therefore$ $f(x)$ is not diffrentiable at $x = 3$ 1½ m

L. H. D at $x = 4$ $\lim_{x\to 4^-} \dfrac{f(x)-f(4)}{x-4}$

$= \dfrac{1-1}{x-4} = 0$

R. H. D at $x = 4$ $\lim_{x\to 4^+} \dfrac{f(x)-f(4)}{x-4}$

$$\lim_{x\to 4^+} \frac{2x-7-1}{x-4} = 2$$

L. H. D at x = 4 $\neq$ R.H.D at x = 4

f(x) is not differentiable at x = 4 1½ m

9. $y = x^{e^{-x^2}}$

$\log y = e^{-x^2} \log x$ 1 m

Diff. w. r. t x

$$\frac{1}{y}\frac{dy}{dx} = \frac{e^{-x^2}}{x} + \log x\, e^{-x^2}(-2x)$$ 2 m

$$\frac{dy}{dx} = y\left(\frac{e^{-x^2}}{x} - 2x \log x\, e^{-x^2}\right)$$ ½ m

$$= x^{e^{-x^2}} e^{-x^2}\left(\frac{1}{x} - 2x \log x\right)$$ ½ m

OR

$$\log\sqrt{x^2+y^2} = \tan^{-1}\frac{x}{y}$$

Diff. w. r. t. x

$$\frac{1}{2(x^2+y^2)}\left(2x+2y\frac{dy}{dx}\right) = \frac{1}{1+\frac{x^2}{y^2}}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)$$ 2 m

$$\frac{x+y\frac{dy}{dx}}{x^2+y^2} = \frac{y^2}{x^2+y^2}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)$$ 1 m

$$\frac{dy}{dx}(y+x) = y-x$$ ½ m

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$ ½ m

10. $y = \sqrt{x+1} - \sqrt{x-1}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x-1}}$$ 1 m

$$= \frac{\sqrt{x-1}-\sqrt{x+1}}{2\sqrt{x^2-1}}$$ ½ m

$$4\left(x^2-1\right)\left(\frac{dy}{dx}\right)^2 = y^2$$ ½ m

$$4\left(x^2-1\right) 2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + 8x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx}$$ 1 m

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{y}{4}$$ ½ m

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{y}{4} = 0$$ ½ m

11. $\displaystyle\int \frac{1-\cos x}{\cos x\left(1+\cos x\right)}\, dx$

$$= \int \frac{1+\cos x - 2\cos x}{\cos x\left(1+\cos x\right)}\, dx$$ 1½ m

$$\int \frac{dx}{\cos x} - 2\int \frac{dx}{1+\cos x}$$ ½ m

$\int \sec x \, dx - \int \sec^2 \frac{x}{2} \, dx$ 1 m

$\log|\sec x + \tan x| - 2 \tan \frac{x}{2} + c$ 1 m

12.
$$\begin{array}{l} \\ \text{Family A} \\ \text{Family B} \\ \text{Family C} \end{array} \begin{array}{c} \text{M W C} \\ \begin{pmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{pmatrix} \end{array} \begin{array}{c} \text{Expenses} \\ \begin{pmatrix} 200 \\ 150 \\ 200 \end{pmatrix} \end{array} = \begin{array}{c} \text{Family expenses} \\ \begin{pmatrix} 1050 \\ 1150 \\ 2300 \end{pmatrix} \end{array}$$ 2 m

Expenses for family A = ₹ 1050

Expenses for family B = ₹ 1150 1 m

Expenses for family C = ₹ 2300

Any relevant impact 1 m

13. $\tan^{-1}x + \tan^{-1} y = \frac{\pi}{2} - \tan^{-1} z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \cot^{-1}z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}\left(\frac{1}{z}\right)$ as $z > 0$ 1 m

$\frac{x+y}{1-xy} = \frac{1}{z}$ ½ m

$xy + yz + zx = 1$ ½ m

14.
$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$ 1 m

$C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_3$

$$(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix} = 0$$ 2 m

$(a + b + c)\,(ab + bc + ca - a^2 - b^2 - c^2) = 0$

given $a \neq b \neq c$, so $ab + bc + ca - a^2 - b^2 - c^2 \neq 0$ ½ m

$\Rightarrow\ (a + b + c) = 0$ ½ m

15. $\dfrac{x-1}{3} = \dfrac{y-1}{-1} = \dfrac{z+1}{0} = \lambda$ ½ m

$\dfrac{x-4}{2} = \dfrac{y}{0} = \dfrac{z+1}{3} = \mu$ ½ m

$x = 3\lambda + 1,\ y = -\lambda + 1,\ z = -1$ 1 m

$x = 2\mu + 4,\ y = 0,\ z = 3\mu - 1$

At the point of intersection

$\lambda = 1,\ \mu = 0$ 1 m

so $3\lambda + 1 = 4 = 2\mu + 4$ ½

Hence the lines are intersecting

Point of intersection is $(4, 0, -1)$ ½ m

16. Coordinats of Q are $-3\mu + 1,\ \mu - 1,\ 5\mu + 2$ ½ m

D.R's of $\overrightarrow{PQ}$ $-3\mu - 2,\ \mu - 3,\ 5\mu - 4$ 1 m

as $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$

$1(-3\mu-2)-4(\mu-3)+3(5\mu-4)=0$ 1½ m

$\mu=\frac{1}{4}$ 1 m

OR

The D.R's of the line are 2, –6, 4 1 m

mid point of the line 2, 1, –1 1 m

The plane passes through (2, 1, –1) and is perpendicular to the

plane

eqn. : $2(x-2)-6(y-1)+4(z+1)=0$

$x-3y+2z+3=0$ 1 m

Vector from : $\vec{r}\cdot\left(\hat{i}-3\hat{j}+2\hat{k}\right)+3=0$ 1 m

17. No's divisible by 6 .................... 16 1m

No's divisible by 8 .................... 12 1m

No's not divisible by 24 ............. 20 1m

Required probabilty $=\frac{20}{100}=\frac{1}{5}$ 1 m

18. $\int x\sin^{-1}x\,dx$

$\frac{x^2}{2}\sin^{-1}x-\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx$ 1 m

$\frac{x^2}{2}\sin^{-1}x+\frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}\,dx$ ½ m

$$\Rightarrow \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int\sqrt{1-x^2}\,dx - \frac{1}{2}\int\frac{dx}{\sqrt{1-x^2}}\,dx$$ 1 m

$$\frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right) - \frac{1}{2}\sin^{-1}x + c$$ 1½

or $$\frac{x^2}{2}\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1}x + c$$

19. $$\int_0^2\left(x^2 + e^{2x+1}\right)dx$$

$$h = \frac{2}{n}$$ ½ m

$$\int_0^2\left(x^2 + e^{2x+1}\right)dx = \lim_{h\to 0} h\left[f(0) + f(0+h) + f(0+2h) + \ldots\ldots\right.$$

$$\left. + \ldots\ldots\ldots f(0+n-1)h\right]$$ 1 m

$$= \lim_{h\to 0} h\left[h^2\left(1^2 + 2^2 + \ldots\ldots + (n-1)^2\right)\right.$$

$$\left. + e\left(1 + e^{2h} + e^{4h} + \ldots\ldots e^{2(n-1)h}\right)\right]$$ 1 m

$$= \lim_{h\to 0}\frac{(nh)(nh-h)(2nh-h)}{6}$$ ½ m

$$+ \lim_{h\to 0} e.h.\left(\frac{e^{2nh}-1}{e^{2h}-1}\right)$$ ½ m

$$= \frac{8}{3} + \frac{\left(e^4-1\right)e}{2} = \frac{8}{3} + \frac{e^5-e}{2}$$ ½ m

OR

$$\int_0^{\pi} \frac{x \tan x \, dx}{\sec x \operatorname{cosec} x}$$

$$\int_0^{\pi} x \sin^2 x \, dx$$ 1 m

Let $I = \int_0^{\pi} x \sin^2 x \, dx$

$= \int_0^{\pi} (\pi - x) \sin^2 (\pi - x) \, dx$ ½ m

$= \int_0^{\pi} (\pi - x) \sin^2 x \, dx$ ½ m

$2I = \pi \int_0^{\pi} \sin^2 x \, dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} \, dx$ ½ m

$= \frac{\pi}{2} \left[ x - \frac{\sin 2x}{2} \right]_0^{\pi}$ 1 m

$= \frac{\pi^2}{2}$

$I = \frac{\pi^2}{4}$ ½ m

## SECTION - C

20. $y = \frac{x}{1 + x^2}$

$\frac{dy}{dx} = \frac{1 - x^2}{\left(1 + x^2\right)^2}$ 2 m

Let $f(x) = \dfrac{1-x^2}{(1+x^2)^2}$

$f'(x) = 0 \Rightarrow \dfrac{-2x(3-x^2)}{(1+x^2)^3} = 0$

For max or min $x(3-x^2) = 0 \Rightarrow x = 0$ or $x = \pm\sqrt{3}$ 2 m

Calculating $\dfrac{d^2f(x)}{dx^2}$ at $x = 0 < 0$

1 m

at $x = \pm\sqrt{3} > 0$

$\Rightarrow$ $x = 0$ is the point of local maxima

1 m

$\Rightarrow$ the required pt is (0, 0)

21. $\dfrac{dy}{dx} = \dfrac{y^2}{xy - x^2}$

Let $y = vx$, $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ ½ m

$v + x\dfrac{dv}{dx} = \dfrac{v^2}{v-1}$ 1½ m

$x\dfrac{dv}{dx} = \dfrac{v}{v-1}$

$\dfrac{dx}{x} = \left(\dfrac{v-1}{v}\right) dv$ 1½ m

$\int \dfrac{dx}{x} = \int \left(1 - \dfrac{1}{v}\right) dv$

$\log x = v - \log v + c$ 1 m

$\log y = \frac{y}{x} + c$ or $x \log y - y = c x$ 1½ m

OR

$$\sin 2x \frac{dy}{dx} - y = \tan x$$

$$\frac{dy}{dx} - \frac{y}{\sin 2x} = \frac{\tan x}{\sin 2x}$$ 1 m

$$\frac{dy}{dx} - y(\text{cosec } 2x) = \frac{\sec^2 x}{2}$$

$$P = -\text{cosec } 2x, \quad Q = \frac{1}{2} \sec^2 x$$

$$\int P\, dx = -\int \text{cosec } 2x\, dx$$

$$= -\frac{1}{2} \log |\tan x|$$

So $e^{\int P\, dx} = \frac{1}{\sqrt{\tan x}}$ 1½ m

Solution is

$$\frac{y}{\sqrt{\tan x}} = \frac{1}{2} \int \frac{\sec^2 x\, dx}{\sqrt{\tan x}} \quad \left( \begin{array}{c} \sqrt{\tan x} = t \\ \Rightarrow \frac{1}{2} \frac{\sec^2 x\, dx}{\sqrt{\tan x}} = dt \end{array} \right)$$ 1½ m

$$\frac{y}{\sqrt{\tan x}} = \sqrt{\tan x} + c$$ 1 m

Getting $c = 1$ ½ m

$\Rightarrow y = \tan x - \sqrt{\tan x}$ ½ m

22. Eqn. of plane

$(x+y+z-6)+\lambda(2x+3y+4z+5) = 0$ 2 m

it passes through $(1,1,1)$

$-3+14\lambda=0 \Rightarrow \lambda = \frac{3}{14}$ 2 m

Eqn. of plane will be

$20x+23y+26z-69=0$ 1 m

vector from : $\vec{r} \cdot \left(20\hat{i}+23\hat{j}+26\hat{k}\right) = 69$ 1 m

23. For every $a \in A$ , $(a, a) \in R$

$\because |a-a| = 0$ is divisible by 2

$\therefore$ R is reflexive 1 m

For all $a, b \in A$

$(a, b) \in R \Rightarrow |a-b|$ is divisible by 2

$\Rightarrow |b-a|$ is divisible by 2

$\therefore$ $(b, a) \in R$ $\therefore$ R is symmetric 1 m

For all $a, b, c \in A$

$(a, b) \in R \Rightarrow |a-b|$ is divisible by 2

$(b, c) \in R \Rightarrow |b-c|$ is divisible by 2

So, $a-b = \pm 2k$ 1 m

$$\begin{array}{l} b-c = \pm 2\ell \\ \hline a-c = \pm 2m \end{array}$$

$\Rightarrow |a-c|$ is divisible by 2

$\Rightarrow \quad (a, c) \in R$

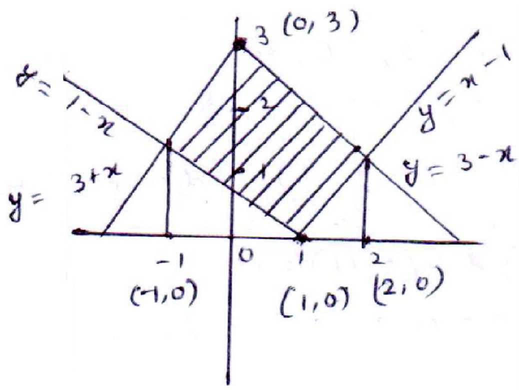
$\Rightarrow$ R is transitive  1 m

Showing elements of $\{1, 3, 5\}$ and  1 m

{2, 4} are related to each other

and $\{1, 3, 5\}$ and $\{2, 4\}$ are not related to each other  1 m

24.




Graph  2 + 2 m

Area of shaded reigon

$$= \int_{-1}^{0} (3 + x + x - 1)\, dx + \int_{0}^{2} (3 - x)\, dx - 2 \int_{1}^{2} (x - 1)\, dx$$  1 m

$$= 2 \left.\frac{(x+1)^2}{2}\right]_{-1}^{0} - \left.\frac{(3-x)^2}{2}\right]_{0}^{2} - 2 \left.\frac{(x-1)^2}{2}\right]_{1}^{2}$$

$$= 1 - \frac{1}{2}(1 - 9) - 1 = 4 \text{ sq. units}$$  1 m

25. Let the no. of items in the item A = x

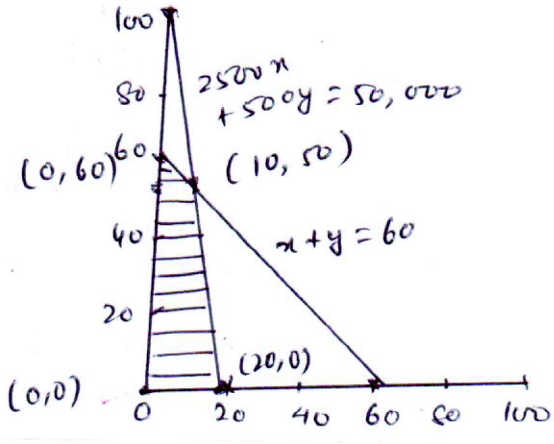
Let the no. of items in the item B = y

(Maximize) $z = 500\,x + 150\,y$  1 m

$x + y \leq 60$

$2500\,x + 500\,y \;\leq\; 50{,}000$ Graph 2 m

$x,\, y \geq 0$




$z(0,0) \;=\; 0$

$z(10,50) \;= 12{,}500$

$z(20, 0) \;=\; 10{,}000$ 2 m

$z(0, 60) \;=\; 9{,}000$

Max. Profit $=$ Rs. 12,500 1 m

OR

Let the no. of packets of food $X \;=\; x$

Let the no. of packets of food $Y \;=\; y$

$P = (6x + 3y)$ (minimize) 1 m

subject to

$12x + 3y \;\geq\; 240$

$4x + 20y \;\geq\; 460$

$6x + 4y \leq\; 300,\;\; x,\, y \geq 0$

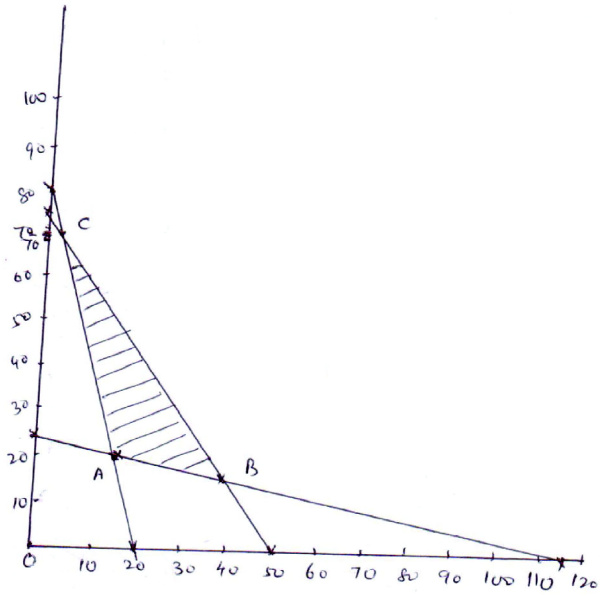
or

$4x + y \;\geq\; 80$

$x + 5y \;\geq\; 115$ 2 m

$3x + 2y \;\leq\; 150$

$x,\, y \geq\; 0$




Correct points of feasible region

A (15, 20), B (40, 151),

C (2, 72)

So P (15, 20) = 150

P (40, 15) = 285

P (2, 72) = 228

Graph 2 m

minimum amount of vitamin A = 150 units when 15 packets of food x and 20 packets of food y are used 1 m

26. Let $E_1$ be the event of following course of meditation and yoga and $E_2$ be the event of following course of drugs 1 m

$$P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{2}$$ 1 m

$$P(A|E_1) = \frac{70 \times 40}{100 \times 100} \quad P(A|E_2) = \frac{75}{100} \times \frac{40}{100}$$ 1 m

Formula 1 m

$$P(E_1|A) = \frac{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100}\right)}{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100} + \frac{1}{2} \times \frac{75}{100}\right)}$$ 2 m

$$= \frac{70}{145} = \frac{14}{29}$$

QUESTION PAPER CODE 65/3/MT

**EXPECTED ANSWERS/VALUE POINTS**

**SECTION - A**

Marks

1. Order : 2, degree : 2, Product : 4 ½ + ½ m

2. $\dfrac{dy}{dx} = -\alpha A \sin \alpha x + \alpha B \cos \alpha x$ ½ m

$$\left.\begin{aligned} \frac{d^2y}{dx^2} &= -\alpha^2 \left(A \cos \alpha x + B \sin \alpha x\right) \\ \frac{d^2y}{dx^2} + \alpha^2 y &= 0 \end{aligned}\right\}$$ ½ m

3. $\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$ or any other correct example ½ + ½ m

4. Value $= 3$ 1 m

5. Writing dr's correctly ½ m

D.C'S $\dfrac{3}{13}$, $\dfrac{4}{13}$, $\dfrac{12}{13}$ ½ m

6. Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|}$ ½ m

Projection $= \dfrac{5}{\sqrt{2}}$ ½ m

## SECTION - B

7. $\int x \sin^{-1}x \, dx$

$\frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}} \, dx$ 1 m

$\frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}} \, dx$ ½ m

$\Rightarrow \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int \sqrt{1-x^2} \, dx - \frac{1}{2}\int \frac{dx}{\sqrt{1-x^2}} \, dx$ 1 m

$\frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right) - \frac{1}{2}\sin^{-1}x + c$ 1½

or $\frac{x^2}{2}\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1}x + c$

8. $\int_0^2 \left(x^2 + e^{2x+1}\right) dx$

$h = \frac{2}{n}$ ½ m

$\int_0^2 \left(x^2 + e^{2x+1}\right) dx = \lim_{h \to 0} h \left[f(0) + f(0+h) + f(0+2h) + \ldots\ldots\right.$

$\left. + \ldots\ldots\ldots f(0+n-1)h \right]$ 1 m

$= \lim_{h \to 0} h \left[h^2\left(1^2 + 2^2 + \ldots\ldots + (n-1)^2\right)\right.$

$\left. + e\left(1 + e^{2h} + e^{4h} + \ldots\ldots e^{2(n-1)h}\right)\right]$ 1 m

$$= \lim_{h \to 0} \frac{(nh)(nh-h)(2nh-h)}{6}$$ ½ m

$$+ \lim_{h \to 0} e.h.\left(\frac{e^{2nh}-1}{e^{2h}-1}\right)$$ ½ m

$$= \frac{8}{3} + \frac{(e^4-1)e}{2} = \frac{8}{3} + \frac{e^5-e}{2}$$ ½ m

OR

$$\int_0^{\pi} \frac{x \tan x \, dx}{\sec x \operatorname{cosec} x}$$

$$\int_0^{\pi} x \sin^2 x \, dx$$ 1 m

$$\text{Let } I = \int_0^{\pi} x \sin^2 x \, dx$$

$$= \int_0^{\pi} (\pi - x) \sin^2 (\pi - x) \, dx$$ ½ m

$$= \int_0^{\pi} (\pi - x) \sin^2 x \, dx$$ ½ m

$$2I = \pi \int_0^{\pi} \sin^2 x \, dx = \pi \int_0^{\pi} \frac{1-\cos 2x}{2} \, dx$$ ½ m

$$= \frac{\pi}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$$ 1 m

$$= \frac{\pi^2}{2}$$

$$I = \frac{\pi^2}{4}$$ ½ m

9. $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$ ½ m

$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu$ ½ m

$x = 3\lambda + 1,\ y = -\lambda + 1,\ z = -1$ 1 m

$x = 2\mu + 4,\ y = 0,\ z = 3\mu - 1$

At the point of intersection

$\lambda = 1,\ \mu = 0$ 1 m

so $3\lambda + 1 = 4 = 2\mu + 4$ ½

Hence the lines are intersecting

Point of intersection is (4, 0, –1) ½ m

10. Coordinats of Q are $-3\mu + 1,\ \mu - 1,\ 5\mu + 2$ ½ m

D.R's of $\overrightarrow{PQ}$ $-3\mu - 2,\ \mu - 3,\ 5\mu - 4$ 1 m

as $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$

$1(-3\mu - 2) - 4(\mu - 3) + 3(5\mu - 4) = 0$ 1½ m

$\mu = \frac{1}{4}$ 1 m

OR

The D.R's of the line are 2, –6, 4 1 m

mid point of the line 2, 1, –1 1 m

The plane passes through (2, 1, –1) and is perpendicular to the

plane

eqn. : $2(x-2) - 6(y-1) + 4(z+1) = 0$

$x - 3y + 2z + 3 = 0$ 1 m

Vector from : $\vec{r} \cdot \left(\hat{i} - 3\hat{j} + 2\hat{k}\right) + 3 = 0$ 1 m

11. No's divisible by 6 .................... 16 1m

No's divisible by 8 .................... 12 1m

No's not divisible by 24 ............. 20 1m

Required probabilty $= \frac{20}{100} = \frac{1}{5}$ 1 m

12. Let $x = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1 m

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$

$$\begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$ 1½ m

$a + 4b = -7, \ c + 4d = 2, \ 2a + 5b = -8, \ 2c + 5d = 4$

Solving $a = 1, \ b = -2, \ c = 2, \ d = 0$ 1 m

$\therefore \ x = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ ½ m

OR

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$|A| = 1 \neq 0$, $A^{-1}$ will exist ½ m

$$\text{adj } A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$ (Any four correct Cofactors : 1 mark) 2 m

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$ ½ m

$$A^{-1} A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ 1 m

13. $f(x) = |x-3| + |x-4|$

$$= \begin{cases} 7-2x, & x < 3 \\ 1, & 3 \le x < 4 \\ 2x-7, & x \ge 4 \end{cases}$$ 1 m

L. H. D at $x = 3$ $\lim\limits_{x \to 3^-} \dfrac{f(x) - f(3)}{x-3}$

$$\lim_{x \to 3^-} \frac{6-2x}{x-3} = -2$$

R. H. D at $x = 3$ $\lim\limits_{x \to 3^+} \dfrac{f(x) - f(3)}{x-3}$

$$= \frac{1-1}{x-3} = 0$$

L.H.D $\neq$ R.H.D $\therefore$ $f(x)$ is not diffrentiable at $x = 3$ 1½ m

L. H. D at $x = 4$ $\lim\limits_{x \to 4^-} \dfrac{f(x) - f(4)}{x - 4}$

$= \dfrac{1-1}{x-4} = 0$

R. H. D at $x = 4$ $\lim\limits_{x \to 4^+} \dfrac{f(x) - f(4)}{x - 4}$

$\lim\limits_{x \to 4^+} \dfrac{2x - 7 - 1}{x - 4} = 2$

L. H. D at $x = 4$ $\neq$ R.H.D at $x = 4$

$f(x)$ is not differentiable at $x = 4$ 1½ m

14. $y = x^{e^{-x^2}}$

$\log y = e^{-x^2} \log x$ 1 m

Diff. w. r. t x

$\dfrac{1}{y} \dfrac{dy}{dx} = \dfrac{e^{-x^2}}{x} + \log x \, e^{-x^2} (-2x)$ 2 m

$\dfrac{dy}{dx} = y\left(\dfrac{e^{-x^2}}{x} - 2x \log x \, e^{-x^2}\right)$ ½ m

$= x^{e^{-x^2}} e^{-x^2} \left(\dfrac{1}{x} - 2x \log x\right)$ ½ m

OR

$\log\sqrt{x^2 + y^2} = \tan^{-1} \dfrac{x}{y}$

Diff. w. r. t. x

$$\frac{1}{2\left(x^2+y^2\right)}\left(2x+2y\frac{dy}{dx}\right) = \frac{1}{1+\frac{x^2}{y^2}}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)$$ 2 m

$$\frac{x+y\frac{dy}{dx}}{x^2+y^2} = \frac{y^2}{x^2+y^2}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)$$ 1 m

$$\frac{dy}{dx}\left(y+x\right) = y-x$$ ½ m

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$ ½ m

15. $y = \sqrt{x+1} - \sqrt{x-1}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x-1}}$$ 1 m

$$= \frac{\sqrt{x-1}-\sqrt{x+1}}{2\sqrt{x^2-1}}$$ ½ m

$$4\left(x^2-1\right)\left(\frac{dy}{dx}\right)^2 = y^2$$ ½ m

$$4\left(x^2-1\right)2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + 8x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx}$$ 1 m

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{y}{4}$$ ½ m

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{y}{4} = 0$$ ½ m

16. $\int \frac{1-\cos x}{\cos x\,(1+\cos x)}\ dx$

$= \int \frac{1+\cos x - 2\cos x}{\cos x\,(1+\cos x)}\ dx$ 1½ m

$\int \frac{dx}{\cos x} - 2\int \frac{dx}{1+\cos x}$ ½ m

$\int \sec x\ dx - \int \sec^2 \frac{x}{2}\ dx$ 1 m

$\log\left|\sec x + \tan x\right| - 2\tan\frac{x}{2} + c$ 1 m

17.

$$\begin{array}{lccc} & \text{M W C} & \text{Expenses} & & \text{Family expenses} \\ \begin{array}{l}\text{Family A}\\ \text{Family B}\\ \text{Family C}\end{array} & \begin{pmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{pmatrix} & \begin{pmatrix} 200 \\ 150 \\ 200 \end{pmatrix} & = & \begin{pmatrix} 1050 \\ 1150 \\ 2300 \end{pmatrix} \end{array}$$ 2 m

Expenses for family A = ₹ 1050

Expenses for family B = ₹ 1150 1 m

Expenses for family C = ₹ 2300

Any relevant impact 1 m

18. $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2} - \tan^{-1}z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \cot^{-1}z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}\left(\frac{1}{z}\right)$ as $z > 0$ 1 m

$$\frac{x+y}{1-xy} = \frac{1}{z}$$ ½ m

$xy + yz + zx = 1$ ½ m

19. $$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$ 1 m

$C_1 \rightarrow C_1 - C_2,\ C_2 \rightarrow C_2 - C_3$

$$(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix} = 0$$ 2 m

$(a + b + c)\ (ab + bc + ca - a^2 - b^2 - c^2) = 0$

given $a \neq b \neq c$, so $ab + bc + ca - a^2 - b^2 - c^2 \neq 0$ ½ m

$\Rightarrow\ (a+b+c) = 0$ ½ m

**SECTION - C**

20. Let $E_1$ be the event of following course of meditation and yoga and $E_2$ be the event of following course of drugs 1 m

$P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{2}$ 1 m

$$P(A|E_1) = \frac{70 \times 40}{100 \times 100} \quad P(A|E_2) = \frac{75}{100} \times \frac{40}{100}$$ 1 m

Formula 1 m

$$P(E_1|A) = \frac{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100}\right)}{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100} + \frac{1}{2} \times \frac{75}{100}\right)}$$

$$= \frac{70}{145} = \frac{14}{29}$$ 2 m

21. Let the no. of items in the item $A = x$

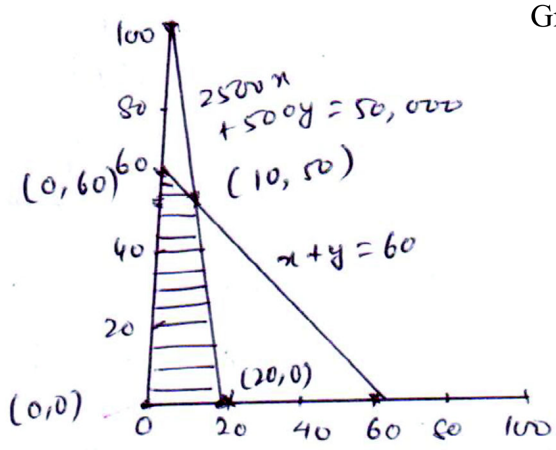
Let the no. of items in the item $B = y$

(Maximize) $z = 500x + 150y$ 1 m

$x + y \le 60$

$2500x + 500y \le 50{,}000$ Graph 2 m

$x, y \ge 0$




$z(0,0) = 0$

$z(10,50) = 12{,}500$

$z(20,0) = 10{,}000$ 2 m

$z(0,60) = 9{,}000$

Max. Profit = Rs. 12,500 1 m

OR

Let the no. of packets of food $X = x$

Let the no. of packets of food $Y = y$

(minimize) $P = (6x + 3y)$ 1 m

subject to

$12x + 3y \geq 240$

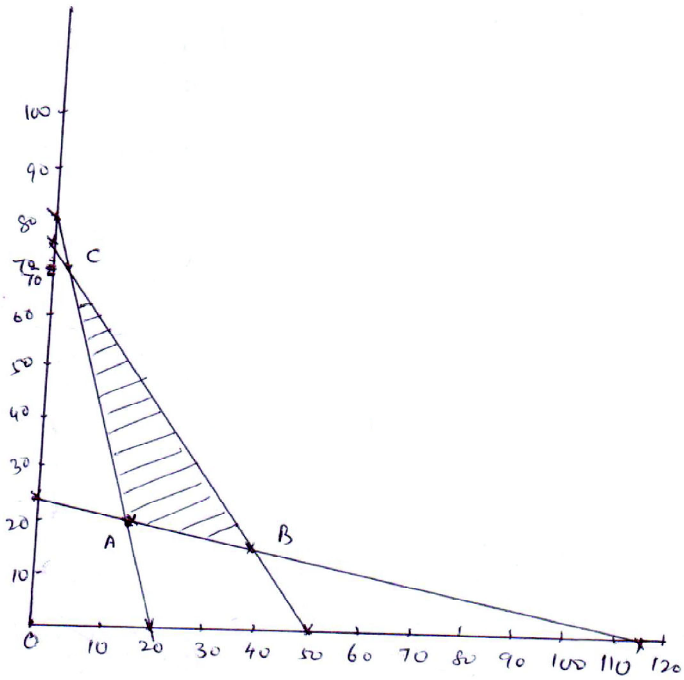
$4x + 20y \geq 460$

$6x + 4y \leq 300, \; x, y \geq 0$

or

$$\left.\begin{array}{l} 4x + y \geq 80 \\ x + 5y \geq 115 \\ 3x + 2y \leq 150 \end{array}\right\} \qquad 2 \text{ m}$$

$x, y \geq 0$




Correct points of feasible region

A (15, 20), B (40, 15), C (2, 72)

So P (15, 20) = 150

P (40, 15) = 285

P (2, 72) = 228

Graph 2 m

minimum amount of vitamin A = 150 units when 15 packets of food X and 20 packets of food Y are used 1 m

22. For every $a \in A$, $(a, a) \in R$

$\because \; |a - a| = 0$ is divisible by 2

$\therefore$ R is reflexive 1 m

For all $a, b \in A$

$(a, b) \in R \Rightarrow |a-b|$ is divisible by 2

$\Rightarrow |b-a|$ is divisible by 2

$\therefore$ $(b, a) \in R$ $\therefore$ R is symmetric 1 m

For all a, b, c $\in$ A

$(a, b) \in R \Rightarrow |a-b|$ is divisible by 2

$(b, c) \in R \Rightarrow |b-c|$ is divisible by 2

So, $a-b = \pm 2k$ 1 m

$$\begin{array}{l} b-c = \pm 2\ell \\ \hline a-c = \pm 2m \end{array}$$

$\Rightarrow |a-c|$ is divisible by 2

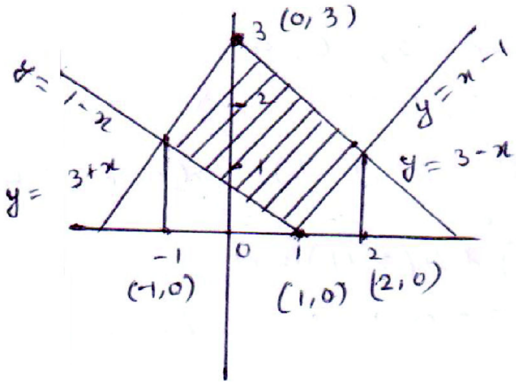
$\Rightarrow (a, c) \in R$

$\Rightarrow$ R is transitive 1 m

Showing elements of $\{1, 3, 5\}$ and 1 m

$\{2, 4\}$ are related to each other

and $\{1, 3, 5\}$ and $\{2, 4\}$ are not related to each other 1 m

23. 


Graph 2 + 2 m

Area of shaded reigon

$$= \int_{-1}^{0} (3+x+x-1)\,dx + \int_{0}^{2} (3-x)\,dx - 2\int_{1}^{2} (x-1)\,dx$$ 1 m

$$= 2\left.\frac{(x+1)^2}{2}\right]_{-1}^{0} - \left.\frac{(3-x)^2}{2}\right]_{0}^{2} - 2\left.\frac{(x-1)^2}{2}\right]_{1}^{2}$$

$$= 1 - \frac{1}{2}(1-9) - 1 = 4 \text{ sq. units}$$ 1 m

24. $$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$$

Let $y = vx$, $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ ½ m

$$v + x\frac{dv}{dx} = \frac{v^2}{v-1}$$ 1½ m

$$x\frac{dv}{dx} = \frac{v}{v-1}$$

$$\frac{dx}{x} = \left(\frac{v-1}{v}\right) dv$$ 1½ m

$$\int\frac{dx}{x} = \int\left(1 - \frac{1}{v}\right)dv$$

$$\log x = v - \log v + c$$ 1 m

$$\log y = \frac{y}{x} + c \text{ or } x\log y - y = cx$$ 1½ m

OR

$$\sin 2x\frac{dy}{dx} - y = \tan x$$

$$\frac{dy}{dx} - \frac{y}{\sin 2x} = \frac{\tan x}{\sin 2x}$$ 1 m

$$\frac{dy}{dx} - y(\operatorname{cosec} 2x) = \frac{\sec^2 x}{2}$$

$P = -\operatorname{cosec} 2x,\ Q = \frac{1}{2}\sec^2 x$

$\int P\,dx = -\int \operatorname{cosec} 2x\,dx$

$= -\frac{1}{2}\log|\tan x|$

So $e^{\int P\,dx} = \frac{1}{\sqrt{\tan x}}$ 1½ m

Solution is

$$\frac{y}{\sqrt{\tan x}} = \frac{1}{2}\int \frac{\sec^2 x\,dx}{\sqrt{\tan x}} \quad \left(\begin{array}{c}\sqrt{\tan x} = t \\ \Rightarrow \frac{1}{2}\frac{\sec^2 x\,dx}{\sqrt{\tan x}} = dt\end{array}\right)$$ 1½ m

$$\frac{y}{\sqrt{\tan x}} = \sqrt{\tan x} + c$$ 1 m

Getting $c = 1$ ½ m

$\Rightarrow y = \tan x - \sqrt{\tan x}$ ½ m

25. Eqn. of plane

$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$ 2 m

it passes through $(1, 1, 1)$

$-3 + 14\lambda = 0 \Rightarrow \lambda = \frac{3}{14}$ 2 m

Eqn. of plane will be

$20x + 23y + 26z - 69 = 0$ 1 m

vector from : $\vec{r} \cdot \left( 20\hat{i} + 23\hat{j} + 26\hat{k} \right) = 69$ 1 m

26. $y = \dfrac{x}{1+x^2}$

$\dfrac{dy}{dx} = \dfrac{1-x^2}{(1+x^2)^2}$ 2 m

Let $f(x) = \dfrac{1-x^2}{(1+x^2)^2}$

$f'(x) = 0 \Rightarrow \dfrac{-2x(3-x^2)}{(1+x^2)^3} = 0$

For max or min $x(3-x^2) = 0 \Rightarrow x = 0$ or $x = \pm\sqrt{3}$ 2 m

Calculating $\dfrac{d^2f(x)}{dx^2}$ at $x = 0 < 0$

1 m

at $x = \pm\sqrt{3} > 0$

$\Rightarrow$ $x = 0$ is the point of local maxima

1 m

$\Rightarrow$ the required pt is (0, 0)