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Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65/1/P, 65/2/P, 65/3/P

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/P

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $|A| = -19$ ½ m

$A^{-1} = -\frac{1}{19}\begin{pmatrix} -2 & -5 \\ -3 & 2 \end{pmatrix}$ ½ m

2. $\frac{dy}{dx} = c$ ½ m

$y = x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^2$ ½ m

3. $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$ ½ m

I.F. $= e^{\tan^{-1}y}$ ½ m

4. $\vec{a}\cdot\left(\vec{b}\times\vec{a}\right) = \left[\vec{a}\ \vec{b}\ \vec{a}\right] = 0$ 1 m

5. $\vec{a} + \vec{b} = 3\hat{i} + 3\hat{j}$ ½ m

$\left(\vec{a} + \vec{b}\right)\cdot\vec{c} = 3$ ½ m

6. $\frac{x+3}{0} = \frac{y-4}{3} = \frac{z-2}{-1}$ ½ m

D.Rs are 0, 3, –1 ½ m

SECTION - B

7. $\begin{matrix} A \\ B \\ C \end{matrix}\begin{pmatrix} 25 & 12 & 34 \\ 22 & 15 & 28 \\ 26 & 18 & 36 \end{pmatrix}\begin{pmatrix} 20 \\ 15 \\ 5 \end{pmatrix}$ 1½ m

$$= \begin{pmatrix} 850 \\ 805 \\ 970 \end{pmatrix}$$ 1½ m

Any relevant value 1 m

8. $$\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left\{\frac{1-\frac{a-b}{a+b}\tan^2\frac{x}{2}}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}\right\}$$ 1½ m

$$= \cos^{-1}\left\{\frac{a+b-a\tan^2\frac{x}{2}+b\tan^2\frac{x}{2}}{a+b+a\tan^2\frac{x}{2}-b\tan^2\frac{x}{2}}\right\}$$ 1 m

$$= \cos^{-1}\left\{\frac{a\left(1-\tan^2\frac{x}{2}\right)+b\left(1+\tan^2\frac{x}{2}\right)}{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}\right\}$$ ½ m

$$= \cos^{-1}\left\{\frac{a\,\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+b}{a+b\,\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\right\}$$ ½ m

$$= \cos^{-1}\left\{\frac{a\cos x+b}{a+b\cos x}\right\}$$ ½ m

OR

$$\tan^{-1}\left(\frac{x-2}{x-3}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-2}{x-3}+\frac{x+2}{x+3}}{1-\frac{x-2}{x-3}\cdot\frac{x+2}{x+3}}\right) = \frac{\pi}{4}$$ 1½ m

$$\Rightarrow \tan^{-1}\left(\frac{2x^2-12}{-5}\right) = \frac{\pi}{4}$$ 1½ m

$$\Rightarrow \frac{2x^2-12}{-5} = 1 \Rightarrow x^2 = \frac{7}{2}$$ ½ m

$$\Rightarrow x = \sqrt{\frac{7}{2}}$$

For writing no solution as $|x| < 1$ ½ m

9. $$A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$ 2 m

$$A^2 - 5A + 16\,I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix} + \begin{pmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{pmatrix}$$ 1 m

$$= \begin{pmatrix} 11 & -1 & -3 \\ -1 & 9 & -10 \\ -5 & 4 & 14 \end{pmatrix}$$ 1 m

10. Taking x from R_2, x (x – 1) from R_3 and (x + 1) from C_3

$$\Delta = x^2 (x-1)(x+1)\begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ -3 & x-2 & 1 \end{vmatrix}$$ 2 m

$C_2 \to C_2 - x\,C_1 ; \quad C_3 \to C_3 - C,$

$$= x^2(x^2-1)\begin{vmatrix} 1 & 0 & 0 \\ 2 & -1-x & -1 \\ -3 & 4x-2 & 4 \end{vmatrix}$$ 1 m

$$= x^2(x^2-1)\begin{vmatrix} -1(1+x) & -1 \\ 4x-2 & 4 \end{vmatrix}$$ ½ m

$$= 6x^2(1-x^2)$$ ½ m

11. $\frac{dx}{dt} = \alpha\left[-2\sin 2t\sin 2t + 2\cos 2t\,(1+\cos 2t)\right]$ 1 m

$\frac{dy}{dt} = \beta\left[2\sin 2t\cos 2t - (1-\cos 2t)\cdot 2\sin 2t)\right]$ 1 m

$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) \Big/ \left(\frac{dx}{dt}\right) = \frac{\beta(2\sin 4t - 2\sin 2t)}{\alpha(2\cos 4t + 2\cos 2t)}$ ½+1 m

$$= \frac{\beta}{\alpha}\cdot\frac{2\cos 3t\sin t}{2\cos 3t\cos t} = \frac{\beta}{\alpha}\tan t$$ ½ m

12. Let y $= \cos^{-1}\left(\frac{x-x^{-1}}{x-x^{-1}}\right) = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$ 1 m

$$= \pi - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$ 1 m

$$= \pi - 2\tan^{-1}x$$ 1 m

$$\therefore \frac{dy}{dx} = -\frac{2}{1+x^2}$$ 1 m

13. Let $y = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} + x^x$

Let $u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\}$; $v = x^x$

$\therefore\ y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ½ m

$u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} = \cos^{-1}\left[\cos\cdot\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right]$ ½ m

$= \frac{\pi}{2} - \sqrt{\frac{1+x}{2}}$

$\therefore\ \frac{du}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}}$ .... (i) ½ m

$v = x^x$

$\therefore\ \log v = x\log x$

$\frac{1}{v}\frac{dv}{dx} = x\cdot\frac{1}{x} + 1\log x = 1 + \log x$

$\frac{dv}{dx} = x^x(1+\log x)$ ....(ii) 1½ m

$\therefore\ \frac{dy}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}} + x^x(1+\log x)$ ½ m

$\left(\frac{dy}{dx}\right)_{\text{at } x=1} = -\frac{1}{4} + 1 = \frac{3}{4}$ ½ m

14. $I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$ .... (i)

$= \int_0^{\pi/2} \frac{2^{\sin\left(\frac{\pi}{2} - x\right)}}{2^{\sin\left(\frac{\pi}{2} - x\right)} + 2^{\cos\left(\frac{\pi}{2} - x\right)}} dx \left[\text{using} \int_0^a (x)\, dx = \int_0^a f(a - x)\, dx\right]$ 1½ m

$= \int_0^{\pi/2} \frac{2^{\cos x}}{2^{\sin x} + 2^{\cos x}} dx$ ....(ii) 1 m

Adding (i) and (ii),

$2I = \int_0^{\pi/2} 1\, dx = [x]_0^{\pi/2} = \frac{\pi}{2}$ 1 m

$\Rightarrow I = \frac{\pi}{4}$ ½ m

OR

$I = \int_0^{3/2} |x \cos(\pi x)|\, dx$

$= \int_0^{1/2} x \cos \pi x\, dx - \int_{1/2}^{3/2} x \cos \pi x\, dx$ 1 m

$= \left[\frac{x \sin \pi x}{\pi}\right]_0^{1/2} - \int_0^{1/2} \frac{\sin \pi x}{\pi} dx - \left[\frac{x \sin \pi x}{\pi}\right]_{1/2}^{3/2} + \int_{1/2}^{3/2} \frac{-\sin \pi x}{\pi} dx$ 1½ m

$= \frac{1}{2\pi} + \frac{1}{\pi^2}[\cos \pi x]_0^{1/2} + \frac{3}{2\pi} + \frac{1}{2\pi} + \frac{1}{\pi^2}[\cos \pi x]_{1/2}^{3/2}$

$= \frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{3}{2\pi} + \frac{1}{2\pi} + 0$ 1 m

$= \frac{5}{2\pi} - \frac{1}{\pi^2}$ ½ m

15. $I = \int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$

$= \int \frac{\cos x + \sin x}{\sqrt{\sin x \cos x}} dx$ 1 m

$= \sqrt{2} \int \frac{(\cos x + \sin x)}{\sqrt{1 - (1 - 2 \sin x \cot x)}} dx$ 1 m

$= \sqrt{2} \int \frac{\cos x + \sin x}{\sqrt{1 - (\sin x - \cos x)^2}} dx$ ½ m

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$ ½ m

$\therefore \; I = \sqrt{2} \int \frac{dt}{\sqrt{1 - t^2}} = \sqrt{2} \sin^{-1} t + c$ ½ m

$= \sqrt{2} \sin^{-1}(\sin x - \cos x) + c$ ½ m

16. $I = \int \frac{x^3 - 1}{x(x^2 + 1)} dx = \int \left(1 - \frac{x + 1}{x(x^2 + 1)}\right) dx$ 1 m

$= x - \int \frac{x + 1}{x(x^2 + 1)} dx$ ½ m

$= x - I_1$

Let $\frac{x + 1}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1} = \frac{1}{x} + \frac{1 - x}{x^2 + 1}$ 1 m

$\therefore \; I_1 = \int \frac{1}{x} + \frac{(1 - x)}{x^2 + 1} dx = \log x - \frac{1}{2} \log \left| x^2 + 1 \right| + \tan^{-1} x$ 1 m

$\therefore \; I = x - \log|x| + \frac{1}{2} \log \left| x^2 + 1 \right| - \tan^{-1} x + c$ ½ m

17. Here $\left.\begin{aligned} \vec{AB} &= -4\hat{i} - 6\hat{j} - 2\hat{k} \\ \vec{AC} &= -\hat{i} + 4\hat{j} + 3\hat{k} \\ \vec{AD} &= -8\hat{i} - \hat{j} + 3\hat{k} \end{aligned}\right\}$ 1½ m

For them to be coplanar, $\left[\vec{AB}\ \vec{AC}\ \vec{AD}\right] = 0$ 1½ m

i.e. $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -60 + 126 - 66 = 0$ ½ m

$\therefore$ Points A, B, C and D are coplanar ½ m

18. Here $\begin{vmatrix} b-c-(a-d) & b-a & b+c-(a+d) \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-\gamma & \beta & \beta+\gamma \end{vmatrix}$ 2½ m

$= 2\begin{vmatrix} b-a & b-a & b+c-a-d \\ \alpha & \alpha & \alpha+\delta \\ \beta & \beta & \beta+\gamma \end{vmatrix}$ $C_1 \to C_1 + C_3$ ½ m

$= 0$ $(\because C_1 \text{ and } C_2 \text{ are identical})$ ½ m

Hence given lines are coplanar ½ m

OR

D.R's of normal to the plane are $5, -4, 7$ 1 m

D.R's of y – axis : $0, 1, 0$ ½ m

If θ is the angle between the plane and y–axis, then

$$\sin\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}$$ 1 m

$$= \frac{-4}{3\sqrt{10}}$$ 1 m

$\therefore\ \theta = \sin^{-1}\left(\frac{-4}{3\sqrt{10}}\right)$

$\therefore$ Acute angle is $\sin^{-1}\left(\frac{4}{3\sqrt{10}}\right)$ ½ m

19. Let E be the event of getting number greater than 4

$\therefore\ P(E) = \frac{1}{3}$ and $P(\bar{E}) = \frac{2}{3}$ ½+½ m

Required Probability = P $(\bar{E}\,E \text{ or } \bar{E}\,\bar{E}\,\bar{E}\,E \text{ or } \bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,E \text{ or } ....)$ 1 m

$= \frac{2}{3}\cdot\frac{1}{3} + \left(\frac{2}{3}\right)^3\cdot\frac{1}{3} + \left(\frac{2}{3}\right)^5\cdot\frac{1}{3} + ...\infty$ 1 m

$= \frac{2}{9}\left[1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^4 + ...\infty\right]$ ½ m

$= \frac{2}{9}\times\frac{9}{5} = \frac{2}{5}$ ½ m

OR

A = {(5, 6, 1), (5, 6, 2), (5, 6, 3), (5, 6, 4), (5, 6, 5), (5, 6, 6),}

$P(A) = \frac{6}{6\times6\times6} = \frac{1}{36}$, P (B) = P (getting 3 or 4 on the third throw) 1½ m

$A\cap B$ = {(5, 6, 3), (5, 6, 4)} $\Rightarrow P(A\cap B) = \frac{2}{6\times6\times6} = \frac{1}{108}$ 1½ m

$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{1}{3}$ 1 m

SECTION - C

20. Let y = (fog) (x) [say y = h (x)]

$= f[g(x)] = f(x^3 + 5)$ 2½ m

$= 2\,(x^3 + 5) - 3$

$= 2\,x^3 + 7$ 2½ m

$\therefore\ \ x = \sqrt[3]{\dfrac{y-7}{2}} = h^{-1}(y)$ ½ m

$\therefore\ \ (fog)^{-1} = \sqrt[3]{\dfrac{x-7}{2}}$ ½ m

OR

Let (x, y) be the identity element in Q × Q, then

$(a, b) * (x, y) = (a, b) = (x, y) * (a, b)\ \ \forall\ (a, b) \in Q \times Q$ 1½ m

$\Rightarrow\ \ (ax, b + ay) = (a, b)$

$\Rightarrow\ \ a = ax\ \ \text{and}\ \ b = b + ay$

$\Rightarrow\ \ x = 1\ \ \text{and}\ \ y = 0$ 1 m

$\therefore$ (1, 0) is the identity element in Q × Q ½ m

Let (a, b) be the invertible element in Q × Q, then

there exists $(\alpha, \beta) \in Q \times Q$ such that

$(a, b) * (\alpha, \beta) = (\alpha, \beta) * (a, b) = (1, 0)$ 1½ m

$\Rightarrow\ \ (a\alpha, b + a\beta) = (1, 0)$ 1 m

$\Rightarrow\ \ \alpha = \dfrac{1}{a},\ \beta = -\dfrac{b}{a}$

$\therefore$ the invertible element in A is $\left(\dfrac{1}{a},\ -\dfrac{b}{a}\right)$ ½ m

21. $f(x) = 2x^3 - 9\,m\,x^2 + 12\,m^2\,x + 1,\ m > 0$

$f'(x) = 6x^2 - 18\,m\,x + 12\,m^2$ 1 m

$f''(x) = 12x - 18\,m$ 1 m

For Max. or minimum, $f'(x) = 0 \Rightarrow 6x^2 - 18\,m\,x + 12\,m^2 = 0$

$$\Rightarrow (x - 2\,m)\,(x - m) = 0$$

$$\Rightarrow x = m \text{ or } 2\,m$$ 1 m

At $x = m$, $f''(x) = 12m - 18m = -\,ve \Rightarrow x = m$ is a maxima 1 m

At $x = 2\,m$, $f''(x) = 24m - 18m = +\,ve \Rightarrow x = 2m$ is manimum 1 m

$\therefore\ p = m$ and $q = 2\,m$ ½ m

Given $p^2 = q \Rightarrow m^2 = 2\,m \Rightarrow m^2 - 2\,m = 0$

$$\Rightarrow m = 0, 2$$

$$\Rightarrow m = 2 \text{ as } m > 0$$ ½ m

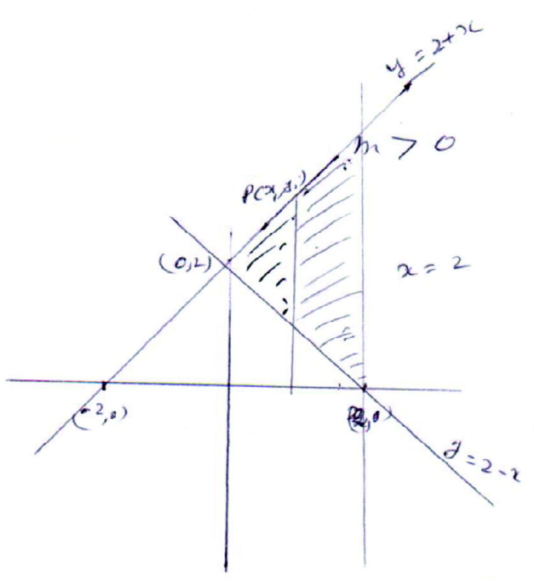
22. $y = 2 + x$ (i)

$y = 2 - x$ (ii)

$x = 2$ (iii),

y_1 is the value of y from (i)

and y_2 is the value of y from (ii)



Required Area $= \int_0^2 (y_1 - y_2)\,dx$ 1 m

correct graph 1+1+1 m

$= \int_0^2 \{(2 + x) - (2 - x)\}\,dx$ correct shading 1 m

$= 2\int_0^2 x\,dx = 2\left[\dfrac{x^2}{2}\right]_0^2$ ½ m

$= 4$ sq. units ½ m

23. Let the equation of line be $y = mx + c$ 1½ m

the line is at unit distance from the origin

i.e. $\left|\frac{0+c}{\sqrt{1+m^2}}\right| = 1 \Rightarrow c = \sqrt{1+m^2}$ 1½ m

$\therefore \quad y = m\,x + \sqrt{1+m^2}$ .... (i) 1 m

$\frac{dy}{dx} = m$ 1 m

$\therefore \quad y = x\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ 1 m

OR

$\frac{dy}{dx} = \frac{x^2+3y^2}{2\,xy} = \frac{1+3\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$ ....(i) 1 m

Differential equation is homogeneous

Put $y = v\,x$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1½ m

$\therefore \quad v + x\frac{dv}{dx} = \frac{1+3v^2}{2v}$ 1 m

$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{2v}$

$\Rightarrow \int\left(\frac{2v}{1+v^2}\right)dv = \int\frac{dx}{x}$ 1 m

$\Rightarrow \log\left|1+v^2\right| = \log|x| + \log c$ 1 m

$\Rightarrow 1+v^2 = c\,x$

$\Rightarrow 1+\left(\frac{y}{x}\right)^2 = c\,x \quad \text{or} \quad x^2+y^2 = c\,x^3$ ½ m

24. Equation of plane passing through (1, 0, 0)

$a(x-1) + b(y-0) + c(z-0) = 0$

or $ax + by + cz - a = 0$ .... (i) 1 m

Plane (i) passes throgh (0, 1, 0)

$b - a = 0$ ....(ii) ½ m

Angle between plane (i) and plane $x + y = 3$ is $\frac{\pi}{4}$ ½ m

$\therefore \quad \cos\frac{\pi}{4} = \frac{a+b}{\sqrt{a^2+b^2+c^2}\sqrt{2}}$ 1 m

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{a+b}{\sqrt{a^2+b^2+c^2}\sqrt{2}}$ 1 m

$\Rightarrow a + b = \sqrt{a^2+b^2+c^2}$

$\Rightarrow 2a = \sqrt{2a^2+c^2}$ (using ii)

$\Rightarrow c = \pm\sqrt{2}\,a$ .... (iii) 1 m

$\therefore$ Equation (i) becomes

$a(x-1) + a(y-0) \pm \sqrt{2}\,a(z-0) = 0$

$\Rightarrow x + y \pm \sqrt{2}\,z - 1 = 0$ ½ m

D.R's of the normal is $1, 1, \pm\sqrt{2}$ ½ m

25. Let E_1, E_2 and E be the events such that

E_1 : students residing in hostel

E_2 : students residing outside hostel 1½ m

E_3 : students getting 'A' grade

$\therefore \quad P(E_1) = \frac{40}{100}, \quad P(E/E_1) = \frac{50}{100}$

$P(E_2) = \frac{60}{100}, \quad P(E/E_2) = \frac{30}{100}$ } 2 m

$$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2)\ \ P(E/E_2)}$$ 1 m

$$= \frac{\frac{40}{100} \times \frac{50}{100}}{\frac{40}{100} \times \frac{50}{100} + \frac{30}{100} \times \frac{60}{100}}$$ 1 m

$$= \frac{10}{19}$$ ½ m

26. Let x be the man helpers and y be the woman helpers

Pay roll : $Z = 225x + 200y$ 1 m

Subject to constraints :

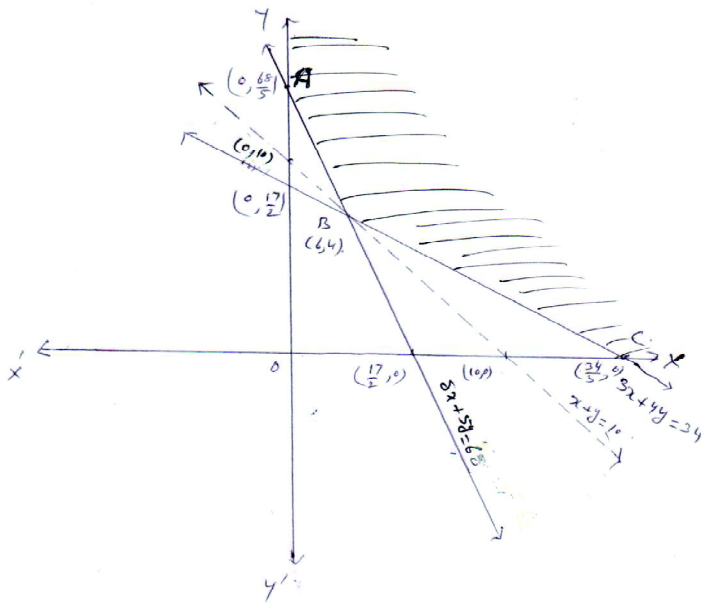
$x + y \leq 10$

$3x + 4y \geq 34$

$8x + 5y \geq 68$

$x \geq 0, y \geq 0$ } ½×4 =2 m

correct graph : 2 m



At A $\left(0, \frac{68}{5}\right)$, Z (A) = Rs. 2720

At B $(6, 4)$, Z (B) = Rs. 2150 Minimum ½ m

At C $\left(\frac{34}{5}, 0\right)$, Z (C) = Rs. 2550

Minimum Z = Rs. 2150 at (6, 4) ½ m

[Feasible region is unbounded and to check minimum

of Z, $225x + 200\,y < 2150$

corresponding line is outside of the shaded region]

QUESTION PAPER CODE 65/2/P

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\vec{a}\cdot(\vec{b}\times\vec{a}) = [\vec{a}\ \vec{b}\ \vec{a}] = 0$ 1 m

2. $\vec{a}+\vec{b} = 3\hat{i}+3\hat{j}$ ½ m

$(\vec{a}+\vec{b})\cdot\vec{c} = 3$ ½ m

3. $\frac{x+3}{0} = \frac{y-4}{3} = \frac{z-2}{-1}$ ½ m

D.Rs are 0, 3, –1 ½ m

4. $|A| = -19$ ½ m

$A^{-1} = -\frac{1}{19}\begin{pmatrix}-2 & -5\\ -3 & 2\end{pmatrix}$ ½ m

5. $\frac{dy}{dx} = c$ ½ m

$y = x\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^2$ ½ m

6. $\frac{dx}{dy}+\frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$ ½ m

I.F. $= e^{\tan^{-1}y}$ ½ m

SECTION - B

7. Let y $= \cos^{-1}\left(\frac{x-x^{-1}}{x-x^{-1}}\right) = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$ 1 m

$= \pi - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ 1 m

$= \pi - 2\tan^{-1}x$ 1 m

$\therefore \frac{dy}{dx} = -\frac{2}{1+x^2}$ 1 m

8. Let $y = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} + x^x$

Let $u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\}; \quad v = x^x$

$\therefore \quad y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ½ m

$u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} = \cos^{-1}\left[\cos\cdot\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right]$ ½ m

$= \frac{\pi}{2} - \sqrt{\frac{1+x}{2}}$

$\therefore \frac{du}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}}$ .... (i) ½ m

$v = x^x$

$\therefore \log v = x \log x$

$\frac{1}{v}\frac{dv}{dx} = x\cdot\frac{1}{x} + 1\log x = 1 + \log x$

$$\frac{dv}{dx} = x^x\left(1+\log x\right) \text{ ....(ii)}$$ 1½ m

$$\therefore \quad \frac{dy}{dx} = -\frac{1}{2\sqrt{2}\,\sqrt{1+x}} + x^x\left(1+\log x\right)$$ ½ m

$$\left(\frac{dy}{dx}\right)_{\text{at } x=1} = -\frac{1}{4}+1=\frac{3}{4}$$ ½ m

9. $$I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}\,dx \text{ .... (i)}$$

$$= \int_0^{\pi/2} \frac{2^{\sin\left(\frac{\pi}{2}-x\right)}}{2^{\sin\left(\frac{\pi}{2}-x\right)}+2^{\cos\left(\frac{\pi}{2}-x\right)}}\,dx \left[\text{using} \int_0^a (x)\,dx = \int_0^a f(a-x)\,dx\right]$$ 1½ m

$$= \int_0^{\pi/2} \frac{2^{\cos x}}{2^{\sin x}+2^{\cos x}}\,dx \text{ ....(ii)}$$ 1 m

Adding (i) and (ii),

$$2I = \int_0^{\pi/2} 1\,dx = \left[x\right]_0^{\pi/2} = \frac{\pi}{2}$$ 1 m

$$\Rightarrow I = \frac{\pi}{4}$$ ½ m

OR

$$I = \int_0^{3/2} \left|\, x\cos(\pi x)\,\right| dx$$

$$= \int_0^{1/2} x\cos\pi x\,dx - \int_{1/2}^{3/2} x\cos\pi x\,dx$$ 1 m

$$= \left[\frac{x\sin\pi x}{\pi}\right]_0^{1/2} - \int_0^{1/2}\frac{\sin\pi x}{\pi}\,dx - \left[\frac{x\sin\pi x}{\pi}\right]_{1/2}^{3/2} + \int_{1/2}^{3/2}\frac{-\sin\pi x}{\pi}\,dx$$ 1½ m

$$= \frac{1}{2\pi} + \frac{1}{\pi^2}\left[\cos \pi x\right]_0^{1/2} + \frac{3}{2\pi} + \frac{1}{2\pi} + \frac{1}{\pi^2}\left[\cos \pi x\right]_{1/2}^{3/2}$$

$$= \frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{3}{2\pi} + \frac{1}{2\pi} + 0$$ 1 m

$$= \frac{5}{2\pi} - \frac{1}{\pi^2}$$ ½ m

10. $\begin{matrix} A \\ B \\ C \end{matrix}\begin{pmatrix} 25 & 12 & 34 \\ 22 & 15 & 28 \\ 26 & 18 & 36 \end{pmatrix}\begin{pmatrix} 20 \\ 15 \\ 5 \end{pmatrix}$ 1½ m

$$= \begin{pmatrix} 850 \\ 805 \\ 970 \end{pmatrix}$$ 1½ m

Any relevant value 1 m

11. $\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left\{\frac{1-\frac{a-b}{a+b}\tan^2\frac{x}{2}}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}\right\}$ 1½ m

$$= \cos^{-1}\left\{\frac{a+b-a\tan^2\frac{x}{2}+b\tan^2\frac{x}{2}}{a+b+a\tan^2\frac{x}{2}-b\tan^2\frac{x}{2}}\right\}$$ 1 m

$$= \cos^{-1}\left\{\frac{a\left(1-\tan^2\frac{x}{2}\right)+b\left(1+\tan^2\frac{x}{2}\right)}{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}\right\}$$ ½ m

$$= \cos^{-1}\left\{\frac{a\,\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+b}{a+b\,\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\right\}$$ ½ m

$$= \cos^{-1}\left\{\frac{a\cos x+b}{a+b\cos x}\right\}$$ ½ m

OR

$$\tan^{-1}\left(\frac{x-2}{x-3}\right)+\tan^{-1}\left(\frac{x+2}{x+3}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-2}{x-3}+\frac{x+2}{x+3}}{1-\frac{x-2}{x-3}\cdot\frac{x+2}{x+3}}\right)=\frac{\pi}{4}$$ 1½ m

$$\Rightarrow \tan^{-1}\left(\frac{2x^2-12}{-5}\right)=\frac{\pi}{4}$$ 1½ m

$$\Rightarrow \frac{2x^2-12}{-5}=1 \Rightarrow x^2=\frac{7}{2}$$ ½ m

$$\Rightarrow x=\sqrt{\frac{7}{2}}$$

For writing no solution as $|x|<1$ ½ m

12. $$A^2=\begin{pmatrix}2&0&1\\2&1&3\\1&-1&0\end{pmatrix}\begin{pmatrix}2&0&1\\2&1&3\\1&-1&0\end{pmatrix}=\begin{pmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{pmatrix}$$ 2 m

$$A^2 - 5A + 16\,I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix} + \begin{pmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{pmatrix}$$ 1 m

$$= \begin{pmatrix} 11 & -1 & -3 \\ -1 & 9 & -10 \\ -5 & 4 & 14 \end{pmatrix}$$ 1 m

13. Taking x from R_2, x (x – 1) from R_3 and (x + 1) from C_3

$$\Delta = x^2 (x-1)(x+1) \begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ -3 & x-2 & 1 \end{vmatrix}$$ 2 m

$$C_2 \to C_2 - x\,C_1\ ;\quad C_3 \to C_3 - C,$$

$$= x^2 (x^2 - 1) \begin{vmatrix} 1 & 0 & 0 \\ 2 & -1-x & -1 \\ -3 & 4x-2 & 4 \end{vmatrix}$$ 1 m

$$= x^2 (x^2 - 1) \begin{vmatrix} -1(1+x) & -1 \\ 4x-2 & 4 \end{vmatrix}$$ ½ m

$$= 6x^2 (1 - x^2)$$ ½ m

14. $$\frac{dx}{dt} = \alpha\left[-2 \sin 2t \sin 2t + 2 \cos 2t\,(1 + \cos 2t)\right]$$ 1 m

$$\frac{dy}{dt} = \beta\left[2 \sin 2t \cos 2t - (1 - \cos 2t) \cdot 2 \sin 2t)\right]$$ 1 m

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) \Big/ \left(\frac{dx}{dt}\right) = \frac{\beta\left(2 \sin 4t - 2 \sin 2t\right)}{\alpha\left(2 \cos 4t + 2 \cos 2t\right)}$$ ½+1 m

$$= \frac{\beta}{\alpha} \cdot \frac{2 \cos 3t \sin t}{2 \cos 3t \cos t} = \frac{\beta}{\alpha} \tan t$$ ½ m

15. Here $\begin{vmatrix} b-c-(a-d) & b-a & b+c-(a+d) \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-\gamma & \beta & \beta+\gamma \end{vmatrix}$ 2½ m

$= 2\begin{vmatrix} b-a & b-a & b+c-a-d \\ \alpha & \alpha & \alpha+\delta \\ \beta & \beta & \beta+\gamma \end{vmatrix}$ $C_1 \to C_1 + C_3$ ½ m

$= 0$ $(\therefore\ C_1$ and C_2 are identical$)$ ½ m

Hence given lines are coplanar ½ m

OR

D.R's of normal to the plane are $5, -4, 7$ 1 m

D.R's of y – axis : $0, 1, 0$ ½ m

If θ is the angle between the plane and y–axis, then

$$\sin\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$ 1 m

$$= \frac{-4}{3\sqrt{10}}$$ 1 m

$$\therefore\ \theta = \sin^{-1}\left(\frac{-4}{3\sqrt{10}}\right)$$

$\therefore$ Acute angle is $\sin^{-1}\left(\frac{4}{3\sqrt{10}}\right)$ ½ m

16. Let E be the event of getting number greater than 4

$\therefore\ P(E) = \frac{1}{3}$ and $P(\bar{E}) = \frac{2}{3}$ ½+½ m

Required Probability $= P\ (\bar{E}\ E$ or $\bar{E}\ \bar{E}\ \bar{E}\ E$ or $\bar{E}\ \bar{E}\ \bar{E}\ \bar{E}\ \bar{E}\ E$ or ....) 1 m

$$= \frac{2}{3} \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^3 \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^5 \cdot \frac{1}{3} + \ldots\ldots\ldots\ldots \infty$$ 1 m

$$= \frac{2}{9}\left[1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^4 + \ldots\ldots\ldots\ldots \infty\right]$$ ½ m

$$= \frac{2}{9} \times \frac{9}{5} = \frac{2}{5}$$ ½ m

OR

A = {(5, 6, 1), (5, 6, 2), (5, 6, 3), (5, 6, 4), (5, 6, 5), (5, 6, 6),}

$P(A) = \frac{6}{6 \times 6 \times 6} = \frac{1}{36}$, P (B) = P (getting 3 or 4 on the third throw) 1½ m

$A \cap B = \{(5, 6, 3), (5, 6, 4)\} \Rightarrow P(A \cap B) = \frac{2}{6 \times 6 \times 6} = \frac{1}{108}$ 1½ m

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{3}$ 1 m

17. $I = \int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$

$$= \int \frac{\cos x + \sin x}{\sqrt{\sin x \cos x}} dx$$ 1 m

$$= \sqrt{2} \int \frac{(\cos x + \sin x)}{\sqrt{1 - (1 - 2 \sin x \cot x)}} dx$$ 1 m

$$= \sqrt{2} \int \frac{\cos x + \sin x}{\sqrt{1 - (\sin x - \cos x)^2}} dx$$ ½ m

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$ ½ m

$$\therefore I = \sqrt{2} \int \frac{dt}{\sqrt{1 - t^2}} = \sqrt{2} \sin^{-1} t + c$$ ½ m

$$= \sqrt{2} \sin^{-1}(\sin x - \cos x) + c$$ ½ m

18. $I = \int \frac{x^3 - 1}{x(x^2+1)} dx = \int \left(1 - \frac{x+1}{x(x^2+1)}\right) dx$ 1 m

$= x - \int \frac{x+1}{x(x^2+1)} dx$ ½ m

$= x - I_1$

Let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx + C}{x^2+1} = \frac{1}{x} + \frac{1-x}{x^2+1}$ 1 m

$\therefore\ I_1 = \int \frac{1}{x} + \frac{(1-x)}{x^2+1} dx = \log x - \frac{1}{2} \log \left| x^2 + 1 \right| + \tan^{-1}x$ 1 m

$\therefore\ I = x - \log |x| + \frac{1}{2} \log \left| x^2 + 1 \right| - \tan^{-1}x + c$ ½ m

19. Here $\left.\begin{aligned} \vec{AB} &= -4\hat{i} - 6\hat{j} - 2\hat{k} \\ \vec{AC} &= -\hat{i} + 4\hat{j} + 3\hat{k} \\ \vec{AD} &= -8\hat{i} - \hat{j} + 3\hat{k} \end{aligned}\right\}$ 1½ m

For them to be coplanar, $\left[\vec{AB}\ \vec{AC}\ \vec{AD}\right] = 0$ 1½ m

i.e. $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -60 + 126 - 66 = 0$ ½ m

$\therefore$ Points A, B, C and D are coplanar ½ m

SECTION - C

20. Let the equation of line be $y = mx + c$ 1½ m

the line is at unit distance from the origin i.e. $\left| \frac{0+c}{\sqrt{1+m^2}} \right| = 1 \Rightarrow c = \sqrt{1+m^2}$ 1½ m

$\therefore \quad y = m\,x + \sqrt{1+m^2}$ .... (i) 1 m

$\frac{dy}{dx} = m$ 1 m

$\therefore \quad y = x\,\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ 1 m

OR

$\frac{dy}{dx} = \frac{x^2+3y^2}{2\,xy} = \frac{1+3\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$ ....(i) 1 m

Differential equation is homogeneous

Put $y = v\,x$

$\Rightarrow \frac{dy}{dx} = v + x\,\frac{dv}{dx}$ 1½ m

$\therefore \quad v + x\,\frac{dv}{dx} = \frac{1+3v^2}{2v}$ 1 m

$\Rightarrow \quad x\,\frac{dv}{dx} = \frac{1+v^2}{2v}$

$\Rightarrow \int \left(\frac{2v}{1+v^2}\right) dv = \int \frac{dx}{x}$ 1 m

$\Rightarrow \log\left|1+v^2\right| = \log|x| + \log c$ 1 m

$\Rightarrow 1+v^2 = c\,x$

$\Rightarrow 1+\left(\frac{y}{x}\right)^2 = c\,x \quad \text{or} \quad x^2+y^2 = c\,x^3$ ½ m

21. Equation of plane passing through (1, 0, 0)

$a(x-1)+b(y-0)+c(z-0)=0$

or $ax+by+cz-a=0$ .... (i) 1 m

Plane (i) passes throgh (0, 1, 0)

$b-a = 0$ ....(ii) ½ m

Angle between plane (i) and plane $x+y=3$ is $\frac{\pi}{4}$ ½ m

$\therefore \quad \cos\frac{\pi}{4}=\frac{a+b}{\sqrt{a^2+b^2+c^2}\sqrt{2}}$ 1 m

$\Rightarrow \frac{1}{\sqrt{2}}=\frac{a+b}{\sqrt{a^2+b^2+c^2}\sqrt{2}}$ 1 m

$\Rightarrow a+b=\sqrt{a^2+b^2+c^2}$

$\Rightarrow 2a=\sqrt{2a^2+c^2}$ (using ii)

$\Rightarrow c=\pm\sqrt{2}\,a$ .... (iii) 1 m

$\therefore$ Equation (i) becomes

$a(x-1)+a(y-0)\pm\sqrt{2}\,a(z-0)=0$

$\Rightarrow x+y\pm\sqrt{2}\,z-1 = 0$ ½ m

D.R's of the normal is $1, 1, \pm\sqrt{2}$ ½ m

22. Let $y=(fog)(x)$ [say $y=h(x)$]

$= f[g(x)] = f(x^3+5)$ 2½ m

$= 2(x^3+5)-3$

$= 2x^3+7$ 2½ m

$\therefore\ x = \sqrt[3]{\frac{y-7}{2}} = h^{-1}(y)$ ½ m

$\therefore\ (fog)^{-1} = \sqrt[3]{\frac{x-7}{2}}$ ½ m

OR

Let (x, y) be the identity element in Q × Q, then

$(a, b) * (x, y) = (a, b) = (x, y) * (a, b)\ \forall\ (a, b) \in Q \times Q$ 1½ m

$\Rightarrow\ (ax, b + ay) = (a, b)$

$\Rightarrow\ a = ax$ and $b = b + ay$

$\Rightarrow\ x = 1$ and $y = 0$ 1 m

$\therefore$ (1, 0) is the identity element in Q × Q ½ m

Let (a, b) be the invertible element in Q × Q, then

there exists $(\alpha, \beta) \in Q \times Q$ such that

$(a, b) * (\alpha, \beta) = (\alpha, \beta) * (a, b) = (1, 0)$ 1½ m

$\Rightarrow\ (a\alpha, b + a\beta) = (1, 0)$ 1 m

$\Rightarrow\ \alpha = \frac{1}{a},\ \beta = -\frac{b}{a}$

$\therefore$ the invertible element in A is $\left(\frac{1}{a}, -\frac{b}{a}\right)$ ½ m

23. $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1,\ m > 0$

$f'(x) = 6x^2 - 18mx + 12m^2$ 1 m

$f''(x) = 12x - 18m$ 1 m

For Max. or minimum, $f'(x) = 0 \Rightarrow 6x^2 - 18mx + 12m^2 = 0$

$\Rightarrow (x - 2m)(x - m) = 0$

$\Rightarrow\ x = m$ or $2m$ 1 m

At x = m, $f''(x) = 12m - 18m = -$ ve $\Rightarrow$ x = m is a maxima 1 m

At x = 2 m, $f''(x) = 24m - 18m = +$ ve $\Rightarrow$ x = 2m is manimum 1 m

$\therefore$ p = m and q = 2 m ½ m

Given $p^2 = q \Rightarrow m^2 = 2m \Rightarrow m^2 - 2m = 0$

$\Rightarrow m = 0, 2$

$\Rightarrow m = 2$ as $m > 0$ ½ m

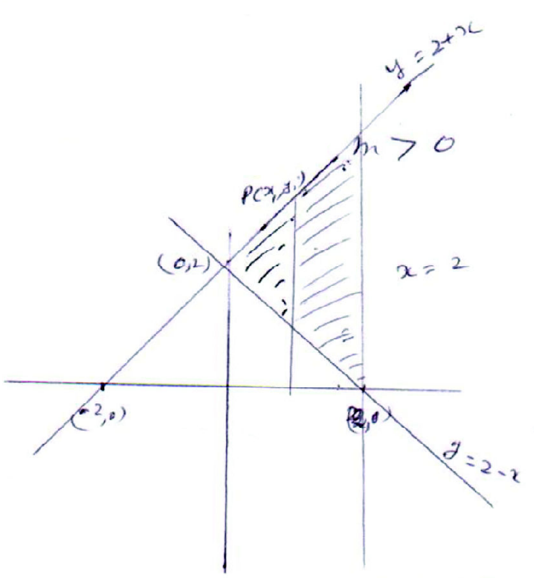
24. $y = 2 + x$ (i)

$y = 2 - x$ (ii)

$x = 2$ (iii),

y_1 is the value of y from (i)

and y_2 is the value of y from (ii)



Required Area $= \int_0^2 (y_1 - y_2)\, dx$ 1 m

correct graph 1+1+1 m

$= \int_0^2 \{(2 + x) - (2 - x)\}\, dx$ correct shading 1 m

$= 2\int_0^2 x\, dx = 2\left[\frac{x^2}{2}\right]_0^2$ ½ m

$= 4$ sq. units ½ m

25. Let x be the man helpers and y be the woman helpers

Pay roll : $Z = 225x + 200y$ 1 m

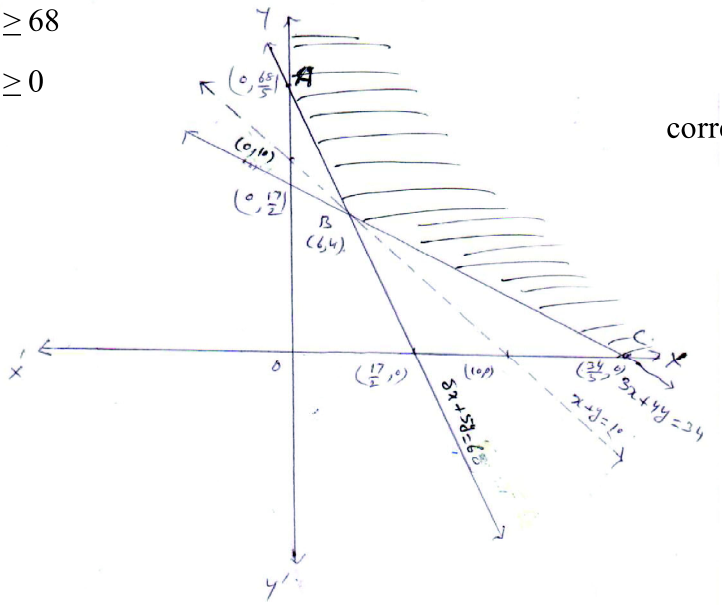
Subject to constraints :

$x + y \leq 10$

$3x + 4y \geq 34$ ½×4 =2 m

$8x + 5y \geq 68$

$x \geq 0, y \geq 0$



correct graph : 2 m

At A $\left(0, \frac{68}{5}\right)$, Z (A) = Rs. 2720

At B $(6, 4)$, Z (B) = Rs. 2150 Minimum ½ m

At C $\left(\frac{34}{5}, 0\right)$, Z (C) = Rs. 2550

Minimum Z = Rs. 2150 at (6, 4) ½ m

[Feasible region is unbounded and to check minimum

of Z, $225x + 200\,y < 2150$

corresponding line is outside of the shaded region]

26. Let E_1, E_2 and E be the events such that

E_1 : students residing in hostel

E_2 : students residing outside hostel 1½ m

E_3 : students getting 'A' grade

$\therefore \quad P(E_1) = \frac{40}{100}, \quad P(E/E_1) = \frac{50}{100}$

2 m

$P(E_2) = \frac{60}{100}, \quad P(E/E_2) = \frac{30}{100}$

$$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \; P(E/E_2)}$$ 1 m

$$= \frac{\frac{40}{100} \times \frac{50}{100}}{\frac{40}{100} \times \frac{50}{100} + \frac{30}{100} \times \frac{60}{100}}$$ 1 m

$$= \frac{10}{19}$$ ½ m

QUESTION PAPER CODE 65/3/P

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$ ½ m

I.F. $= e^{\tan^{-1}y}$ ½ m

2. $|A| = -19$ ½ m

$A^{-1} = -\frac{1}{19}\begin{pmatrix} -2 & -5 \\ -3 & 2 \end{pmatrix}$ ½ m

3. $\frac{dy}{dx} = c$ ½ m

$y = x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^2$ ½ m

4. $\frac{x+3}{0} = \frac{y-4}{3} = \frac{z-2}{-1}$ ½ m

D.Rs are 0, 3, –1 ½ m

5. $\vec{a}\cdot\left(\vec{b}\times\vec{a}\right) = \left[\vec{a}\ \vec{b}\ \vec{a}\right] = 0$ 1 m

6. $\vec{a}+\vec{b} = 3\hat{i}+3\hat{j}$ ½ m

$\left(\vec{a}+\vec{b}\right)\cdot\vec{c} = 3$ ½ m

SECTION - B

7. Here $\left.\begin{aligned} \vec{AB} &= -4\hat{i}-6\hat{j}-2\hat{k} \\ \vec{AC} &= -\hat{i}+4\hat{j}+3\hat{k} \\ \vec{AD} &= -8\hat{i}-\hat{j}+3\hat{k} \end{aligned}\right\}$ 1½ m

For them to be coplanar, $\left[\vec{AB}\ \ \vec{AC}\ \ \vec{AD}\right] = 0$ 1½ m

i.e. $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -60 + 126 - 66 = 0$ ½ m

$\therefore$ Points A, B, C and D are coplanar ½ m

8. Here $\begin{vmatrix} b - c - (a - d) & b - a & b + c - (a + d) \\ \alpha - \delta & \alpha & \alpha + \delta \\ \beta - \gamma & \beta & \beta + \gamma \end{vmatrix}$ 2½ m

$= 2\begin{vmatrix} b - a & b - a & b + c - a - d \\ \alpha & \alpha & \alpha + \delta \\ \beta & \beta & \beta + \gamma \end{vmatrix} \quad C_1 \to C_1 + C_3$ ½ m

$= 0 \qquad (\because\ C_1 \text{ and } C_2 \text{ are identical})$ ½ m

Hence given lines are coplanar ½ m

OR

D.R's of normal to the plane are $5, -4, 7$ 1 m

D.R's of y – axis : $0, 1, 0$ ½ m

If θ is the angle between the plane and y–axis, then

$$\sin\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}$$ 1 m

$$= \frac{-4}{3\sqrt{10}}$$ 1 m

$$\therefore\ \theta = \sin^{-1}\left(\frac{-4}{3\sqrt{10}}\right)$$

$\therefore$ Acute angle is $\sin^{-1}\left(\frac{4}{3\sqrt{10}}\right)$ ½ m

9. Let E be the event of getting number greater than 4

$\therefore \quad P(E) = \frac{1}{3} \quad$ and $\quad P(\bar{E}) = \frac{2}{3}$ ½+½ m

Required Probability $= P\ (\bar{E}\,E \text{ or } \bar{E}\,\bar{E}\,\bar{E}\,E \text{ or } \bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,E \text{ or } ....)$ 1 m

$$= \frac{2}{3}\cdot\frac{1}{3} + \left(\frac{2}{3}\right)^3\cdot\frac{1}{3} + \left(\frac{2}{3}\right)^5\cdot\frac{1}{3} + ...\infty$$ 1 m

$$= \frac{2}{9}\left[1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^4 + ...\infty\right]$$ ½ m

$$= \frac{2}{9}\times\frac{9}{5} = \frac{2}{5}$$ ½ m

OR

A = {(5, 6, 1), (5, 6, 2), (5, 6, 3), (5, 6, 4), (5, 6, 5), (5, 6, 6),}

$P(A) = \frac{6}{6\times6\times6} = \frac{1}{36}$, P (B) = P (getting 3 or 4 on the third throw) 1½ m

$A \cap B = \{(5, 6, 3), (5, 6, 4)\} \Rightarrow P(A \cap B) = \frac{2}{6\times6\times6} = \frac{1}{108}$ 1½ m

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{3}$ 1 m

10. Let $y = \cos^{-1}\left(\frac{x - x^{-1}}{x - x^{-1}}\right) = \cos^{-1}\left(\frac{x^2 - 1}{x^2 + 1}\right)$ 1 m

$$= \pi - \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$$ 1 m

$$= \pi - 2\tan^{-1}x$$ 1 m

$$\therefore \frac{dy}{dx} = -\frac{2}{1 + x^2}$$ 1 m

11. Let $y = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} + x^x$

Let $u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\}$; $v = x^x$

$\therefore\ y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ½ m

$u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} = \cos^{-1}\left[\cos\cdot\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right]$ ½ m

$= \frac{\pi}{2} - \sqrt{\frac{1+x}{2}}$

$\therefore\ \frac{du}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}}$ .... (i) ½ m

$v = x^x$

$\therefore\ \log v = x \log x$

$\frac{1}{v}\frac{dv}{dx} = x\cdot\frac{1}{x} + 1\log x = 1 + \log x$

$\frac{dv}{dx} = x^x\left(1 + \log x\right)$ ....(ii) 1½ m

$\therefore\ \frac{dy}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}} + x^x\left(1 + \log x\right)$ ½ m

$\left(\frac{dy}{dx}\right)_{at\ x=1} = -\frac{1}{4} + 1 = \frac{3}{4}$ ½ m

12. $I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$ .... (i)

$= \int_0^{\pi/2} \frac{2^{\sin\left(\frac{\pi}{2} - x\right)}}{2^{\sin\left(\frac{\pi}{2} - x\right)} + 2^{\cos\left(\frac{\pi}{2} - x\right)}} dx \left[\text{using} \int_0^a (x)\, dx = \int_0^a f(a - x)\, dx\right]$ 1½ m

$= \int_0^{\pi/2} \frac{2^{\cos x}}{2^{\sin x} + 2^{\cos x}} dx$ ....(ii) 1 m

Adding (i) and (ii),

$2I = \int_0^{\pi/2} 1\, dx = [x]_0^{\pi/2} = \frac{\pi}{2}$ 1 m

$\Rightarrow I = \frac{\pi}{4}$ ½ m

OR

$I = \int_0^{3/2} |x \cos(\pi x)|\, dx$

$= \int_0^{1/2} x \cos \pi x\, dx - \int_{1/2}^{3/2} x \cos \pi x\, dx$ 1 m

$= \left[\frac{x \sin \pi x}{\pi}\right]_0^{1/2} - \int_0^{1/2} \frac{\sin \pi x}{\pi} dx - \left[\frac{x \sin \pi x}{\pi}\right]_{1/2}^{3/2} + \int_{1/2}^{3/2} \frac{-\sin \pi x}{\pi} dx$ 1½ m

$= \frac{1}{2\pi} + \frac{1}{\pi^2} [\cos \pi x]_0^{1/2} + \frac{3}{2\pi} + \frac{1}{2\pi} + \frac{1}{\pi^2} [\cos \pi x]_{1/2}^{3/2}$

$= \frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{3}{2\pi} + \frac{1}{2\pi} + 0$ 1 m

$= \frac{5}{2\pi} - \frac{1}{\pi^2}$ ½ m

13. $I = \int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$

$= \int \frac{\cos x + \sin x}{\sqrt{\sin x \cos x}} dx$ 1 m

$= \sqrt{2} \int \frac{(\cos x + \sin x)}{\sqrt{1-(1-2\sin x \cot x)}} dx$ 1 m

$= \sqrt{2} \int \frac{\cos x + \sin x}{\sqrt{1-(\sin x - \cos x)^2}} dx$ ½ m

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$ ½ m

$\therefore \; I = \sqrt{2} \int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2} \sin^{-1} t + c$ ½ m

$= \sqrt{2} \sin^{-1} (\sin x - \cos x) + c$ ½ m

14. $I = \int \frac{x^3 - 1}{x(x^2+1)} dx = \int \left(1 - \frac{x+1}{x(x^2+1)}\right) dx$ 1 m

$= x - \int \frac{x+1}{x(x^2+1)} dx$ ½ m

$= x - I_1$

Let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx + C}{x^2+1} = \frac{1}{x} + \frac{1-x}{x^2+1}$ 1 m

$\therefore \; I_1 = \int \frac{1}{x} + \frac{(1-x)}{x^2+1} dx = \log x - \frac{1}{2} \log |x^2+1| + \tan^{-1} x$ 1 m

$\therefore \; I = x - \log|x| + \frac{1}{2} \log |x^2+1| - \tan^{-1} x + c$ ½ m

15. $\begin{matrix} A \\ B \\ C \end{matrix}\begin{pmatrix} 25 & 12 & 34 \\ 22 & 15 & 28 \\ 26 & 18 & 36 \end{pmatrix}\begin{pmatrix} 20 \\ 15 \\ 5 \end{pmatrix}$ 1½ m

$= \begin{pmatrix} 850 \\ 805 \\ 970 \end{pmatrix}$ 1½ m

Any relevant value 1 m

16. $\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left\{\frac{1-\frac{a-b}{a+b}\tan^2\frac{x}{2}}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}\right\}$ 1½ m

$= \cos^{-1}\left\{\frac{a+b-a\tan^2\frac{x}{2}+b\tan^2\frac{x}{2}}{a+b+a\tan^2\frac{x}{2}-b\tan^2\frac{x}{2}}\right\}$ 1 m

$= \cos^{-1}\left\{\frac{a\left(1-\tan^2\frac{x}{2}\right)+b\left(1+\tan^2\frac{x}{2}\right)}{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}\right\}$ ½ m

$= \cos^{-1}\left\{\frac{a\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+b}{a+b\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\right\}$ ½ m

$= \cos^{-1}\left\{\frac{a\cos x+b}{a+b\cos x}\right\}$ ½ m

OR

$$\tan^{-1}\left(\frac{x-2}{x-3}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-2}{x-3}+\frac{x+2}{x+3}}{1-\frac{x-2}{x-3}\cdot\frac{x+2}{x+3}}\right) = \frac{\pi}{4}$$ 1½ m

$$\Rightarrow \tan^{-1}\left(\frac{2x^2-12}{-5}\right) = \frac{\pi}{4}$$ 1½ m

$$\Rightarrow \frac{2x^2-12}{-5} = 1 \Rightarrow x^2 = \frac{7}{2}$$ ½ m

$$\Rightarrow x = \sqrt{\frac{7}{2}}$$

For writing no solution as $|x| < 1$ ½ m

17. $$A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$ 2 m

$$A^2 - 5A + 16\,I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix} + \begin{pmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{pmatrix}$$ 1 m

$$= \begin{pmatrix} 11 & -1 & -3 \\ -1 & 9 & -10 \\ -5 & 4 & 14 \end{pmatrix}$$ 1 m

18. Taking x from R_2, x (x – 1) from R_3 and (x + 1) from C_3

$$\Delta = x^2 (x-1)(x+1) \begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ -3 & x-2 & 1 \end{vmatrix}$$ 2 m

$C_2 \to C_2 - x\, C_1$; $C_3 \to C_3 - C,$

$$= x^2 (x^2 - 1) \begin{vmatrix} 1 & 0 & 0 \\ 2 & -1-x & -1 \\ -3 & 4x-2 & 4 \end{vmatrix}$$ 1 m

$$= x^2 (x^2 - 1) \begin{vmatrix} -1\,(1+x) & -1 \\ 4x-2 & 4 \end{vmatrix}$$ ½ m

$$= 6x^2 (1 - x^2)$$ ½ m

19. $$\frac{dx}{dt} = \alpha\left[-2 \sin 2t \sin 2t + 2 \cos 2t\,(1 + \cos 2t)\right]$$ 1 m

$$\frac{dy}{dt} = \beta\left[2 \sin 2t \cos 2t - (1 - \cos 2t) \cdot 2 \sin 2t)\right]$$ 1 m

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) \Big/ \left(\frac{dx}{dt}\right) = \frac{\beta\left(2 \sin 4t - 2 \sin 2t\right)}{\alpha\left(2 \cos 4t + 2 \cos 2t\right)}$$ ½+1 m

$$= \frac{\beta}{\alpha} \cdot \frac{2 \cos 3t \sin t}{2 \cos 3t \cos t} = \frac{\beta}{\alpha} \tan t$$ ½ m

SECTION - C

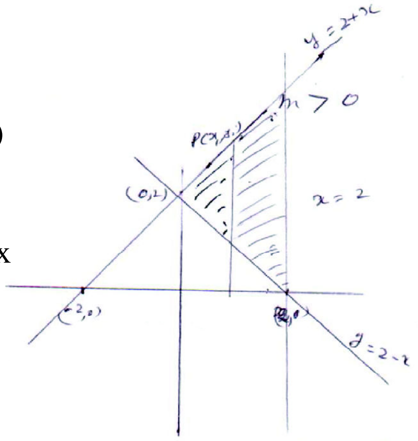
20. $y = 2 + x$ (i)

$y = 2 - x$ (ii)

$x = 2$ (iii),

y_1 is the value of y from (i)

and y_2 is the value of y from (ii)



Required Area $= \int_0^2 (y_1 - y_2)\,dx$ 1 m

correct graph 1+1+1 m

$= \int_0^2 \{(2+x)-(2-x)\}\,dx$ correct shading 1 m

$= 2\int_0^2 x\,dx = 2\left[\frac{x^2}{2}\right]_0^2$ ½ m

$= 4$ sq. units ½ m

21. Let the equation of line be $y = mx + c$ 1½ m

the line is at unit distance from the origin

i.e. $\left|\frac{0+c}{\sqrt{1+m^2}}\right| = 1 \Rightarrow c = \sqrt{1+m^2}$ 1½ m

$\therefore \quad y = mx + \sqrt{1+m^2}$ .... (i) 1 m

$\frac{dy}{dx} = m$ 1 m

$\therefore \quad y = x\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ 1 m

OR

$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy} = \frac{1+3\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$ ....(i) 1 m

Differential equation is homogeneous

Put y = v x

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ 1½ m

$$\therefore \quad v + x\frac{dv}{dx} = \frac{1+3v^2}{2v}$$ 1 m

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\Rightarrow \int \left(\frac{2v}{1+v^2}\right) dv = \int \frac{dx}{x}$$ 1 m

$$\Rightarrow \log\left|1+v^2\right| = \log|x| + \log c$$ 1 m

$$\Rightarrow 1+v^2 = c\,x$$

$$\Rightarrow 1+\left(\frac{y}{x}\right)^2 = c\,x \quad \text{or} \quad x^2 + y^2 = c\,x^3$$ ½ m

22. Let x be the man helpers and y be the woman helpers

Pay roll : Z = 225 x + 200 y 1 m

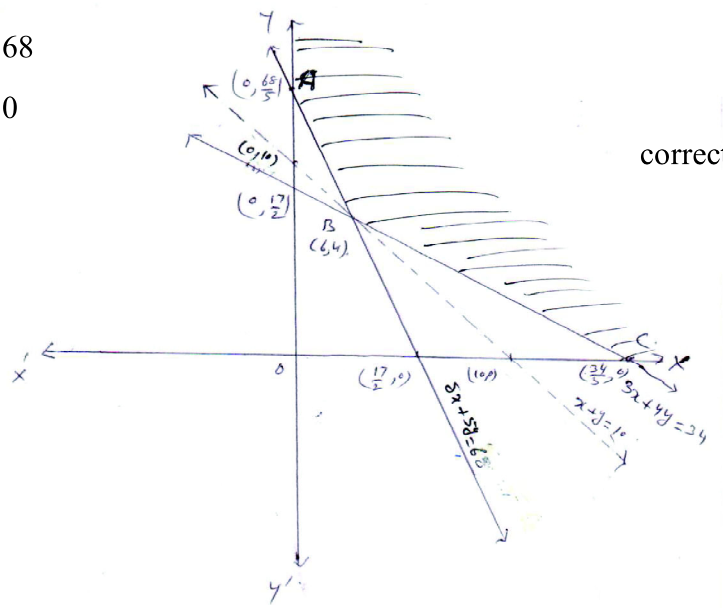
Subject to constraints :

$x + y \leq 10$

$3x + 4y \geq 34$ ½×4 =2 m

$8x + 5y \geq 68$

$x \geq 0,\ y \geq 0$



correct graph : 2 m

At A $\left(0, \frac{68}{5}\right)$, Z (A) = Rs. 2720

At B $(6, 4)$, Z (B) = Rs. 2150 Minimum ½ m

At C $\left(\frac{34}{5}, 0\right)$, Z (C) = Rs. 2550

Minimum Z = Rs. 2150 at (6, 4) ½ m

[Feasible region is unbounded and to check minimum

of Z, $225x + 200y < 2150$

corresponding line is outside of the shaded region]

23. Equation of plane passing through (1, 0, 0)

$a(x-1) + b(y-0) + c(z-0) = 0$

or $ax + by + cz - a = 0$ .... (i) 1 m

Plane (i) passes throgh (0, 1, 0)

$b - a = 0$ ....(ii) ½ m

Angle between plane (i) and plane $x + y = 3$ is $\frac{\pi}{4}$ ½ m

$\therefore \quad \cos\frac{\pi}{4} = \frac{a+b}{\sqrt{a^2+b^2+c^2}\ \sqrt{2}}$ 1 m

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{a+b}{\sqrt{a^2+b^2+c^2}\ \sqrt{2}}$ 1 m

$\Rightarrow a + b = \sqrt{a^2+b^2+c^2}$

$\Rightarrow 2a = \sqrt{2a^2+c^2}$ (using ii)

$\Rightarrow \quad c = \pm\sqrt{2}\, a$ .... (iii) 1 m

$\therefore$ Equation (i) becomes

$a(x-1) + a(y-0) \pm \sqrt{2}\, a(z-0) = 0$

$\Rightarrow \quad x + y \pm \sqrt{2}\, z - 1 = 0$ ½ m

D.R's of the normal is $1, 1, \pm\sqrt{2}$ ½ m

24. Let E_1, E_2 and E be the events such that

E_1 : students residing in hostel

E_2 : students residing outside hostel 1½ m

E_3 : students getting 'A' grade

$\therefore \quad P(E_1) = \frac{40}{100}, \quad P(E/E_1) = \frac{50}{100}$

2 m

$P(E_2) = \frac{60}{100}, \quad P(E/E_2) = \frac{30}{100}$

$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2)\ P(E/E_2)}$ 1 m

$= \frac{\frac{40}{100} \times \frac{50}{100}}{\frac{40}{100} \times \frac{50}{100} + \frac{30}{100} \times \frac{60}{100}}$ 1 m

$= \frac{10}{19}$ ½ m

25. Let $y = (fog)(x)$ [say $y = h(x)$]

$= f[g(x)] = f(x^3 + 5)$ 2½ m

$= 2(x^3 + 5) - 3$

$= 2x^3 + 7$ 2½ m

$\therefore \quad x = \sqrt[3]{\frac{y-7}{2}} = h^{-1}(y)$ ½ m

$\therefore \quad (fog)^{-1} = \sqrt[3]{\frac{x-7}{2}}$ ½ m

OR

Let (x, y) be the identity element in Q × Q, then

$(a, b) * (x, y) = (a, b) = (x, y) * (a, b) \;\; \forall \;\; (a, b) \in Q \times Q$ 1½ m

$\Rightarrow \quad (ax, b + ay) = (a, b)$

$\Rightarrow \quad a = ax \quad \text{and} \quad b = b + ay$

$\Rightarrow \quad x = 1 \quad \text{and} \quad y = 0$ 1 m

$\therefore$ (1, 0) is the identity element in Q × Q ½ m

Let (a, b) be the invertible element in Q × Q, then

there exists $(\alpha, \beta) \in Q \times Q$ such that

$(a, b) * (\alpha, \beta) = (\alpha, \beta) * (a, b) = (1, 0)$ 1½ m

$\Rightarrow \quad (a\alpha, b + a\beta) = (1, 0)$ 1 m

$\Rightarrow \quad \alpha = \frac{1}{a}, \; \beta = -\frac{b}{a}$

$\therefore$ the invertible element in A is $\left(\frac{1}{a}, \; -\frac{b}{a}\right)$ ½ m

26. $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1, \; m > 0$

$f'(x) = 6x^2 - 18mx + 12m^2$ 1 m

$f''(x) = 12x - 18m$ 1 m

For Max. or minimum, $f'(x) = 0 \Rightarrow 6x^2 - 18mx + 12m^2 = 0$

$\Rightarrow (x - 2m)(x - m) = 0$

$\Rightarrow \; x = m \text{ or } 2m$ 1 m

At $x = m$, $f''(x) = 12m - 18m = -$ ve $\Rightarrow$ $x = m$ is a maxima 1 m

At $x = 2$ m, $f''(x) = 24m - 18m = +$ ve $\Rightarrow$ $x = 2m$ is manimum 1 m

$\therefore$ $p = m$ and $q = 2$ m ½ m

Given $p^2 = q \Rightarrow m^2 = 2m \Rightarrow m^2 - 2m = 0$

$\Rightarrow m = 0, 2$

$\Rightarrow m = 2$ as $m > 0$ ½ m