Strictly Confidential — (For Internal and Restricted Use Only)

Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65/1/C, 65/2/C, 65/3/C

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/C

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\vec{a} \times \vec{b} = -17\hat{i} + 13\hat{j} + 7\hat{k}, \left|\vec{a} \times \vec{b}\right| = \sqrt{507}$ ½+½ m

2. $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}, \ \theta = \frac{2\pi}{3}$ ½ + ½ m

3. $d = \left|\frac{\vec{a} \cdot \vec{n} - p}{|\vec{n}|}\right|, \ \text{distance} = \frac{13}{7}$ ½ + ½ m

4. $e^{2x} \sin 2x$ 1 m

5. $y = mx, \frac{dy}{dx} = \frac{y}{x}$ ½ + ½ m

6. $\frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x}$, Integrating factor $= \log x$ ½ + ½ m

SECTION - B

7. $A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$ 1½ m

$A^2 - 4A - 51 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix} = O$ 1 m

$A^2 - 4A - 51 = O \Rightarrow A^{-1} = \frac{1}{5}(A - 4I)$ 1 m

$$A^{-1} = \frac{1}{5}\left\{\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix} - \begin{bmatrix}4 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 4\end{bmatrix}\right\} = \frac{1}{5}\begin{bmatrix}-3 & 2 & 2\\ 2 & -3 & 2\\ 2 & 2 & -3\end{bmatrix}$$ ½ m

OR

$$\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} A$$ 1 m

Using elementary row operations to reach at

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix} A$$ 2 m

$$A^{-1} = \begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$$ 1 m

8. $$\begin{vmatrix}x+2 & x+6 & x-1\\ x+6 & x-1 & x+2\\ x-1 & x+2 & x+6\end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix}3x+7 & x+6 & x-1\\ 3x+7 & x-1 & x+2\\ 3x+7 & x+2 & x+6\end{vmatrix} = 0$$ 1 m

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\begin{vmatrix}3x+7 & x+6 & x-1\\ 1 & -7 & 3\\ 1 & -4 & 7\end{vmatrix} = 0$$ 2 m

$(3x+7)(-37) = 0 \Rightarrow x = \frac{-7}{3}$ 1 m

9. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\, dx \Rightarrow 2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}\, dx$ 1 m

$$2I = \int_0^{\pi/2} \frac{\sec^2 x/2}{2\tan\frac{x}{2} + 1 - \tan^2 x/2}\, dx$$

$I = -\int_0^1 \frac{1}{(t-1)^2 - (\sqrt{2})^2}\, dt$, where $\tan\frac{x}{2} = t$ 1½ m

$I = \left[-\frac{1}{2\sqrt{2}} \log\left|\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right| \right]_0^1$ 1 m

$I = \frac{1}{2\sqrt{2}} \log\left|\frac{1+\sqrt{2}}{1-\sqrt{2}}\right|$ ½ m

OR

$\int_{-1}^{2} \left(e^{3x} + 7x - 5\right) dx$ here $h = \frac{3}{n}$ ½ m

$= \lim_{h \to 0} h \left[f(-1) + f(-1+h) + \ldots\ldots\ldots\ldots \right]$

$= \lim_{h \to 0} h \left[\left(e^{-3} - 12\right) + \left(e^{-3+3h} + 7h - 12\right) + \ldots\ldots + \left(e^{-3+\overline{n-1}\,h} + 7(n-1)h - 12\right) \right]$ 1 m

$= \lim_{h \to 0} h \left[e^{-3}\left(1 + e^{3h} + e^{6h} + \ldots\ldots + e^{3(n-1)h}\right) + 7h\left(1 + 2 + 3 + \ldots.\overline{n-1}\right) - 12\,nh \right]$ 1 m

$= \lim_{h \to 0} h \left[\frac{e^{-3}\left(e^{3nh} - 1\right)h}{e^{3h} - 1} + \frac{7(nh)(nh-h)}{2} - 12nh \right]$ 1 m

$= \frac{e^{-3}\left(e^9 - 1\right)}{3} + \frac{63}{2} - 36 = \frac{e^9 - 1}{3e^3} - \frac{9}{2}$ ½ m

10. $\int \frac{x^2}{x^4 + x^2 - 2}\,dx$

$\int \frac{x^2}{x^4 + x^2 - 2} = \frac{t}{t^2 + t - 2} = \frac{t}{(t+2)(t-1)}$ where $x^2 = t$ 1½ m

$= \frac{2}{3(t+2)} + \frac{1}{3(t-1)}$ 1½ m

$\int \frac{x^2}{x^4 + x^2 - 2}\,dx = \int \frac{2}{3(x^2+2)}\,dx + \int \frac{1}{3(x^2-1)}\,dx$

$= \frac{2}{3\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{1}{6} \log \left| \frac{x-1}{x+1} \right| + c$ 1 m

11. Let E_1 : two headed coin is chosen

E_2 : unbiased coin is chosen

A : All 5 tosses are heads ½ m

$P(E_1) = \frac{1}{5},\ P(E_2) = \frac{4}{5},\ P\left(A/E_1\right) = 1,\ P\left(A/E_2\right) = \frac{1}{32}$ 2 m

$P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)}$ ½ m

$P\left(E_1/A\right) = \frac{\frac{1}{5}\times 1}{\frac{1}{5}\times 1 + \frac{4}{5}\cdot\frac{1}{32}} = \frac{8}{9}$ 1 m

OR

Let the coin is tossed n times

$1 - P(0) > \frac{80}{100}$ 1½ m

$P(0) < \frac{1}{5}$ ½ m

${}^{n}C_{0}\left(\frac{1}{2}\right)^{n}\left(\frac{1}{2}\right)^{0} < \frac{1}{5}$ 1 m

$\left(\frac{1}{2}\right)^{n} < \frac{1}{5} \Rightarrow n \geq 3$ 1 m

12. $\overrightarrow{BA} = \hat{i} + (x-1)\hat{j} + 4\hat{k}, \overrightarrow{CA} = \hat{i} - 3\hat{k}, \ \overrightarrow{DA}\ 3\hat{i} + 3\hat{j} - 2\hat{k}$ 1½ m

$$\left[\overrightarrow{BA}, \overrightarrow{CA}, \overrightarrow{DA}\right] = 0$$ 1 m

$$\begin{vmatrix} 1 & x-1 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$ 1 m

$x = 4$ ½ m

13. $\vec{r} = (4\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})\ \vec{a} + \lambda\vec{b}$ 1 m

Let L be the foot of perpendicular

Position vector of L is $(2\lambda + 4)\hat{i} + (3\lambda + 2)\hat{j} + (6\lambda + 2)\hat{k}$ ½ m

$\overrightarrow{PL} = (2\lambda + 3)\hat{i} + 3\lambda\hat{j} + (6\lambda - 1)\hat{k}$ ½ m

$\overrightarrow{PL} \cdot \vec{b} = 2(2\lambda+3) + 3(3\lambda) + 6(6\lambda-1) = 0$

$\Rightarrow \lambda = 0$ 1 m

$\overrightarrow{PL} = 3\hat{i} - \hat{k}$

$\left|\overrightarrow{PL}\right| = \sqrt{10}$ units 1 m

14. $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$(1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$ 1 m

$1-x = \cos\left(2\sin^{-1}x\right)$ 1 m

$1-x = 1-2x^2$ 1 m

$\Rightarrow x = 0, \frac{1}{2}$ ½ + ½ m

$x = \frac{1}{2}$ is rejected

OR

L.H.S $= 2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$

$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$ 1 m

$= \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$ 1 m

$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7}\cdot\frac{17}{31}}\right)$ 1 m

$= \tan^{-1}\left(\frac{625}{625}\right) = \frac{\pi}{4}$ 1 m

15. $y = e^{ax} \cos bx$

$y_1 = ae^{ax} \cos bx - b\, e^{ax} \sin b\, x$ 1 m

$y_1 = ay - b\, e^{ax} \sin b\, x$ 1 m

$y_2 = ay_1 - b\, [ae^{ax} \sin bx + b\, e^{ax} \cos b\, x]$ 1 m

$y_2 = ay_1 - a\, be^{ax} \sin bx - b^2\, e^{ax} \cos b\, x$

$y_2 = a\, y_1 - a\, (ay - y_1) - b^2\, y$

$y_2 - 2\, a\, y_1 + (a^2 + b^2)\, y = 0$ 1 m

16. $x^x + x^y + y^x = a^b$

Let $u = x^x$, $v = x^y$, $w = y^x$, $\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ½ m

$\frac{du}{dx} = x^x \left(1 + \log x\right)$ 1 m

$\frac{dv}{dx} = x^y \left(\frac{y}{x} + \frac{dy}{dx} \log x\right)$ 1 m

$\frac{dw}{dx} = y^x \left(\frac{x}{y} \cdot \frac{dy}{dx} + \log y\right)$ 1 m

$\frac{dy}{dx} = -\left(\frac{x^x \left(1 + \log x\right) + y\, x^{y-1} + y^x \log y}{x^y \log x + x\, y^{x-1}}\right)$ ½ m

17. $\frac{dx}{at} = a\ [\sin 2t\,(-2\sin 2t) + (1+\cos 2t)(2\cos 2t)]$ 1 m

$\frac{dy}{dt} = b\ [2\sin 2t\cos 2t - 2\sin 2t\,(1-\cos 2t)]$ 1 m

$\frac{dy}{dx} = \frac{b\ [2\sin 2t\cos 2t - 2\sin 2t\,(1-\cos 2t)]}{a\ [\sin t\,(-2\sin 2t) + (1+\cos 2t)(2\cos 2t)]}$ 1 m

$= \frac{4b\cos 3t\sin t}{4a\cos 3t\cos t} = \frac{b}{a}\tan t = \frac{b}{2}\times 1 = \frac{b}{a}$ 1 m

18. $\int \frac{x+3}{(x+5)^3}\ e^x\,dx$

$\int \frac{1}{(x+5)^2} - \frac{2}{(x+5)^3}\ e^x\,dx$ 1 m

$\int \frac{1}{(x+5)^2}\ e^x\,dx - \int \frac{2}{(x+5)^3}\ e^x\,dx$ ½ m

$= \frac{1}{(x+5)^2}\ e^x + \int \frac{2}{(x+5)^3}\ e^x\,dx - \int \frac{2}{(x+5)^3}\ e^x\,dx$ 2 m

$= \frac{e^x}{(x+5)^2} + c$ ½ m

19. $\begin{matrix} & F & M & T \\ x \\ y \\ z \end{matrix}$ $\begin{matrix} x \\ y \\ z \end{matrix}\begin{pmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{pmatrix}\begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 5450 \\ 5250 \\ 6625 \end{pmatrix}$ 1½ m

Funds collected by school x : ₹ 5450, school y = ₹ 5250

school z = ₹ 6625 1 m

Total collected funds = ₹ 17325 ½ m

For writing any value 1 m

SECTION - C

20. (i) Let (e, e') be the identity element in A

$(a, b) * (e, e') = (a, b) = (e, e') * (a, b)$

$(a e, b + a e') = (a, b)$

$$\left.\begin{matrix} ae = a \Rightarrow e = 1 \\ b + a e' = b \Rightarrow e' = 0 \end{matrix}\right] \Rightarrow \text{identity} : (1, 0)$$ 2 ½ m

(ii) Let (x, y) is inverse of $(a, b) \in A$

$(a, b) * (x, y) = (1, 0) = (x, y) * (a, b)$

$(a x, b + a y) = (1, 0)$

$$\left.\begin{matrix} ax = 1 \Rightarrow x = \frac{1}{a} \\ b + a y = 0 \Rightarrow y = \frac{-b}{a} \end{matrix}\right] \Rightarrow \text{inverse of } (a, b) = \left(\frac{1}{a}, \frac{-b}{a}\right)$$ 2 ½ m

Inverse of $(5, 3) = \left(\frac{1}{5}, \frac{-3}{5}\right)$ ½ m

Inverse of $\left(\frac{1}{2}, 4\right) = (2, -8)$ ½ m

OR

One – One : - Case I : when x and y are even

$f(x) = f(y) \Rightarrow x + 1 = y + 1 \Rightarrow x = y$

Case II : when x and y are odd

$f(x) = f(y) \Rightarrow x - 1 = y - 1 \Rightarrow x = y$

Case III : one of them is even and one of them is odd

$f(x) \neq f(y) \Rightarrow x + 1 \neq y - 1 \Rightarrow x \neq y$ 2 ½ m

Onto : Let $y \in W$

$$f(y-1) = y \text{ if } y \text{ is odd}$$

$$f(y+1) = y \text{ if } y \text{ is even}$$

So $\forall\ y \in W$, there exist some element in domain of f

$\Rightarrow$ f is invertible 2½ m

$$f^{-1}(x) = \begin{cases} x-1, & x \text{ is odd} \\ x+1, & x \text{ is even} \end{cases}$$ 1 m

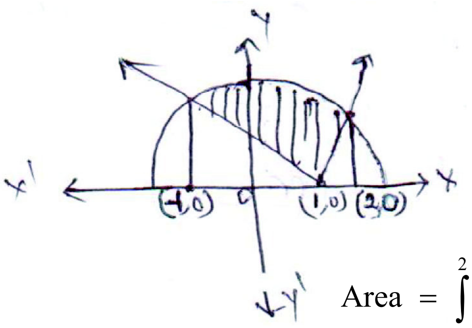
21. 

Figure 1 m

For finding (– 1, 0) , (1, 0) (2, 0) 1½ m

$$\text{Area} = \int_{-1}^{2} \sqrt{5-x^2}\ dx - \int_{-1}^{1} -(x-1)\ dx - \int_{-1}^{2} (x-1)\ dx$$ 1½ m

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} + \left[\frac{(x-1)^2}{2}\right]_{-1}^{1} - \left[\frac{(x-1)^2}{2}\right]_{1}^{2}$$ 1 ½ m

$$= \left(1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right) + \left(1+\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2}\times 4 - \frac{1}{2}\times 1$$

$$= \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2} \text{ sq. units}$$ ½ m

22. $x^2\, dy = (2\, xy + y^2)\, dx$

$$\frac{dy}{dx} = \frac{2\, xy + y^2}{x^2}$$ ½ m

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ 1 m

$v + x\dfrac{dv}{dx} = 2v + v^2 \Rightarrow \int \dfrac{1}{v^2+v}\,dv = \int \dfrac{1}{x}\,dx$ 2 m

$\Rightarrow \log\left|\dfrac{v}{v+1}\right| = \log x + \log c$ 1 m

$\Rightarrow \log\left|\dfrac{y}{y+x}\right| = \log cx \Rightarrow \dfrac{y}{y+x} = cx$ 1 m

$x = 1,\ y = 1 \Rightarrow c = \dfrac{1}{2}$

$x^2 + xy - 2y = 0$ ½ m

OR

Given differential equation can be written as

$\dfrac{dy}{dx} + \dfrac{1}{1+x^2}\,y = \dfrac{e^{m\tan^{-1}x}}{1+x^2}$ 1 m

Integrating factor is $e^{\tan^{-1}x}$ 1 m

Solution is $y \cdot e^{\tan^{-1}x} = \int \dfrac{e^{m\tan^{-1}x}}{1+x^2} \cdot e^{\tan^{-1}x}\,dx$ 1½ m

$\Rightarrow y\,e^{\tan^{-1}x} = \int e^{(m+1)t}dt$, where $\tan^{-1}x = t$ 1 m

$= \dfrac{e^{(m+1)t}}{m+1} = \dfrac{e^{(m+1)\tan^{-1}x}}{m+1} + c$ 1 m

$y = 1, x = 0 \Rightarrow c = \dfrac{m}{m+1}$

$y\,e^{\tan^{-1}x} = \dfrac{e^{(m+1)\tan^{-1}x}}{m+1} + \dfrac{m}{m+1}$ ½ m

23. $f(x) = \sin^2 x - \cos x$

$f'(x) = \sin x\,(2\cos x + 1)$ 1 m

$f'(x) = 0 \Rightarrow \sin x = 0$ and $2\cos x + 1 = 0 \Rightarrow x = 0,\ 2\frac{\pi}{3},\ \pi$ 2½ m

$f(0) = -1,\ f\left(\frac{2\pi}{3}\right) = \frac{5}{4},\ f(\pi) = 1$ 1½ m

Absolute maximum value is $\frac{5}{4}$ ½ m

Absolute minimum value is – 1 ½ m

24. Two lines $\vec{r} = \vec{a_1} + \lambda\,\vec{b_1}$ and $r = \vec{a_2} + \mu\,\vec{b_2}$ are coplanar

if $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = 0$ 1 m

Here $(-\hat{i} + 3\hat{j} + \hat{k}) \cdot [(\hat{i} - \hat{j} + \hat{k}) \times (2\hat{i} - \hat{j} + 3\hat{k})] = 0$ 2 m

Equation of plane is

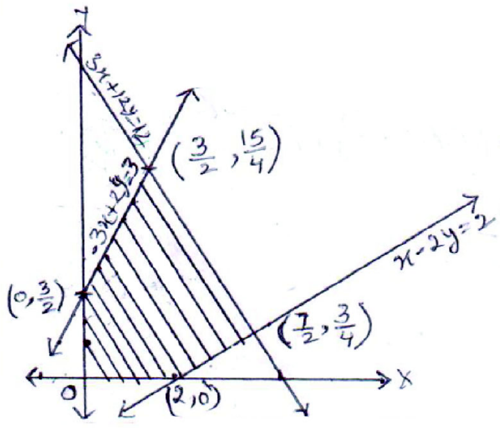
$$(\vec{r} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = 0$$ 1 m

$$\left[\vec{r} - (\hat{i} + \hat{j} + \hat{k})\right] \cdot [(\hat{i} - \hat{j} + \hat{k}) \times (2\hat{i} - \hat{j} + 3\hat{k})] = 0$$

$\vec{r} \cdot (-2\hat{i} - \hat{j} + \hat{k}) + 2 = 0$ 2 m

25. Correct graph of three lines 1×3 m

correct shading of feasible region 1 m



vertices are $\left(0, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{15}{4}\right),$

$\left(\frac{7}{2}, \frac{3}{4}\right), (2, 0)$ 1 m

$z = 5x + 2y$ is maximum

at $\left(\frac{7}{2}, \frac{3}{4}\right) = 19$ and

minimum at $\left(0, \frac{3}{2}\right) = 3$ 1 m

26.

x :	2	3	4	5	6	1 m
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$	2 m
x . P (x) :	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{12}{15}$	$\frac{20}{15}$	$\frac{30}{15}$	½ m
x^2 P (x) :	$\frac{4}{15}$	$\frac{18}{15}$	$\frac{48}{15}$	$\frac{100}{15}$	$\frac{180}{15}$	½ m

Mean $= \sum x \cdot P(x) = \frac{70}{15} = \frac{14}{3}$ 1 m

Variance $= \sum x^2 P(x) = (\text{Mean})^2 = \frac{350}{15} - \frac{196}{9} = \frac{14}{9}$ 1 m

QUESTION PAPER CODE 65/2/C

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1.	$e^{2x} \sin 2x$	1 m
2.	$y = mx, \frac{dy}{dx} = \frac{y}{x}$	½ + ½ m
3.	$\frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x}$, Integrating factor $= \log x$	½ + ½ m
4.	$\vec{a} \times \vec{b} = -17\hat{i} + 13\hat{j} + 7\hat{k}, \left\| \vec{a} \times \vec{b} \right\| = \sqrt{507}$	½+½ m
5.	$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\|\vec{b}\|}$, $\theta = \frac{2\pi}{3}$	½ + ½ m
6.	$d = \left\| \frac{\vec{a} \cdot \vec{n} - p}{\|\vec{n}\|} \right\|$, distance $= \frac{13}{7}$	½ + ½ m

SECTION - B

7.	$\overrightarrow{BA} = \hat{i} + (x-1)\hat{j} + 4\hat{k}, \overrightarrow{CA} = \hat{i} - 3\hat{k}, \overrightarrow{DA}\ 3\hat{i} + 3\hat{j} - 2\hat{k}$	1½ m
	$\left[\overrightarrow{BA}, \overrightarrow{CA}, \overrightarrow{DA}\right] = 0$	1 m
	$\begin{vmatrix} 1 & x-1 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$	1 m
	$x = 4$	½ m

8. $\vec{r} = \left(4\hat{i} + 2\hat{j} + 2\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)\ \vec{a} + \lambda\vec{b}$ 1 m

Let L be the foot of perpendicular

Position vector of L is $(2\lambda + 4)\hat{i} + (3\lambda + 2)\hat{j} + (6\lambda + 2)\hat{k}$ ½ m

$\vec{PL} = (2\lambda + 3)\hat{i} + 3\lambda\hat{j} + (6\lambda - 1)\hat{k}$ ½ m

$\overrightarrow{PL} \cdot \vec{b} = 2(2\lambda + 3) + 3(3\lambda) + 6(6\lambda - 1) = 0$

$\Rightarrow \quad \lambda = 0$ 1 m

$\overrightarrow{PL} = 3\hat{i} - \hat{k}$

$\left|\overrightarrow{PL}\right| = \sqrt{10}$ units 1 m

9. $\sin^{-1}(1 - x) - 2\sin^{-1}x = \frac{\pi}{2}$

$(1 - x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$ 1 m

$1 - x = \cos\left(2\sin^{-1}x\right)$ 1 m

$1 - x = 1 - 2x^2$ 1 m

$\Rightarrow x = 0, \frac{1}{2}$ ½ + ½ m

$x = \frac{1}{2}$ is rejected

OR

L.H.S $= 2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$

$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$ 1 m

$= \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$ 1 m

$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right)$ 1 m

$= \tan^{-1}\left(\frac{625}{625}\right) = \frac{\pi}{4}$ 1 m

10. $A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$ 1½ m

$A^2 - 4A - 51 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix} = O$ 1 m

$A^2 - 4A - 51 = O \Rightarrow A^{-1} = \frac{1}{5}(A - 4I)$ 1 m

$A^{-1} = \frac{1}{5}\left\{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$ ½ m

OR

$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1 m

Using elementary row operations to reach at

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$ 2 m

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ 1 m

11. $$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 3x+7 & x+6 & x-1 \\ 3x+7 & x-1 & x+2 \\ 3x+7 & x+2 & x+6 \end{vmatrix} = 0$$ 1 m

$R_2 \to R_2 - R_1\,,\ R_3 \to R_3 - R_1$

$$\begin{vmatrix} 3x+7 & x+6 & x-1 \\ 1 & -7 & 3 \\ 1 & -4 & 7 \end{vmatrix} = 0$$ 2 m

$(3x+7)(-37) = 0 \Rightarrow x = \frac{-7}{3}$ 1 m

12. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx \Rightarrow 2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}\,dx$ 1 m

$$2I = \int_0^{\pi/2} \frac{\sec^2 x/2}{2\tan\frac{x}{2} + 1 - \tan^2 x/2}\,dx$$

$I = -\int_0^1 \frac{1}{(t-1)^2 - (\sqrt{2})^2}\,dt$, where $\tan\frac{x}{2} = t$ 1½ m

$$I = \left[-\frac{1}{2\sqrt{2}}\log\left|\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right|\right]_0^1$$ 1 m

$$I = \frac{1}{2\sqrt{2}} \log \left| \frac{1+\sqrt{2}}{1-\sqrt{2}} \right|$$ ½ m

OR

$$\int_{-1}^{2} \left(e^{3x} + 7x - 5\right) dx \quad \text{here } h = \frac{3}{n}$$ ½ m

$$= \lim_{h \to 0} h \left[f(-1) + f(-1+h) + \ldots\ldots\ldots\ldots \right]$$

$$= \lim_{h \to 0} h \left[\left(e^{-3} - 12\right) + \left(e^{-3+3h} + 7h - 12\right) + \ldots\ldots + \left(e^{-3+\overline{n-1}\,h} + 7\,(n-1)\,h - 12\right) \right]$$ 1 m

$$= \lim_{h \to 0} h \left[e^{-3}\left(1 + e^{3h} + e^{6h} + \ldots\ldots + e^{3(n-1)h}\right) + 7h\left(1 + 2 + 3 + \ldots.\overline{n-1}\right) - 12\, nh \right]$$ 1 m

$$= \lim_{h \to 0} h \left[\frac{e^{-3}\left(e^{3nh} - 1\right) h}{e^{3h} - 1} + \frac{7\,(nh)(nh - h)}{2} - 12nh \right]$$ 1 m

$$= \frac{e^{-3}\left(e^{9} - 1\right)}{3} + \frac{63}{2} - 36 = \frac{e^{9} - 1}{3\, e^{3}} - \frac{9}{2}$$ ½ m

13. $\int \frac{x^2}{x^4 + x^2 - 2}\, dx$

$$\int \frac{x^2}{x^4 + x^2 - 2} = \frac{t}{t^2 + t - 2} = \frac{t}{(t+2)(t-1)} \quad \text{where } x^2 = t$$ 1½ m

$$= \frac{2}{3\,(t+2)} + \frac{1}{3\,(t-1)}$$ 1½ m

$$\int \frac{x^2}{x^4 + x^2 - 2}\, dx = \int \frac{2}{3\left(x^2 + 2\right)}\, dx + \int \frac{1}{3\left(x^2 - 1\right)}\, dx$$

$$= \frac{2}{3\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{1}{6} \log \left| \frac{x-1}{x+1} \right| + c$$ 1 m

14. Let E_1 : two headed coin is chosen

E_2 : unbiased coin is chosen

A : All 5 tosses are heads ½ m

$$P(E_1) = \frac{1}{5},\ P(E_2) = \frac{4}{5},\ P\left(A/E_1\right) = 1,\ P\left(A/E_2\right) = \frac{1}{32}$$ 2 m

$$P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)}$$ ½ m

$$P\left(E_1/A\right) = \frac{\frac{1}{5}\times 1}{\frac{1}{5}\times 1+\frac{4}{5}\cdot\frac{1}{32}} = \frac{8}{9}$$ 1 m

OR

Let the coin is tossed n times

$$1 - P(0) > \frac{80}{100}$$ 1½ m

$$P(0) < \frac{1}{5}$$ ½ m

$${}^{n}C_0\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^0 < \frac{1}{5}$$ 1 m

$$\left(\frac{1}{2}\right)^n < \frac{1}{5} \Rightarrow n \geq 3$$ 1 m

15. $\int \frac{x+3}{(x+5)^3}\ e^x\ dx$

$$\int \frac{1}{(x+5)^2} - \frac{2}{(x+5)^3}\ e^x\ dx$$ 1 m

$\int \frac{1}{(x+5)^2} e^x dx - \int \frac{2}{(x+5)^3} e^x dx$ ½ m

$= \frac{1}{(x+5)^2} e^x + \int \frac{2}{(x+5)^3} e^x dx - \int \frac{2}{(x+5)^3} e^x dx$ 2 m

$= \frac{e^x}{(x+5)^2} + c$ ½ m

16. $\begin{matrix} & F & M & T \\ x \\ y \\ z \end{matrix} \begin{pmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 5450 \\ 5250 \\ 6625 \end{pmatrix}$ 1½ m

Funds collected by school x : ₹ 5450, school y = ₹ 5250

school z = ₹ 6625 1

Total collected funds = ₹ 17325 ½ m

For writing any value 1 m

17. $y = e^{ax} \cos bx$

$y_1 = ae^{ax} \cos bx - b e^{ax} \sin b x$ 1 m

$y_1 = ay - b e^{ax} \sin b x$ 1 m

$y_2 = ay_1 - b [ae^{ax} \sin bx + b e^{ax} \cos b x]$ 1 m

$y_2 = ay_1 - a be^{ax} \sin bx - b^2 e^{ax} \cos b x$

$y_2 = a y_1 - a (ay - y_1) - b^2 y$

$y_2 - 2 a y_1 + (a^2 + b^2) y = 0$ 1 m

18. $x^x + x^y + y^x = a^b$

Let $u = x^x$, $v = x^y$, $w = y^x$, $\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ½ m

$\frac{du}{dx} = x^x \left(1 + \log x\right)$ 1 m

$\frac{dv}{dx} = x^y \left(\frac{y}{x} + \frac{dy}{dx} \log x\right)$ 1 m

$\frac{dw}{dx} = y^x \left(\frac{x}{y} \cdot \frac{dy}{dx} + \log y\right)$ 1 m

$\frac{dy}{dx} = -\left(\frac{x^x \left(1 + \log x\right) + y\, x^{y-1} + y^x \log y}{x^y \log x + x\, y^{x-1}}\right)$ ½ m

19. $\frac{dx}{at} = a \left[\sin 2t \left(-2 \sin 2t\right) + \left(1 + \cos 2t\right)\left(2 \cos 2t\right)\right]$ 1 m

$\frac{dy}{dt} = b \left[2 \sin 2t \cos 2t - 2 \sin 2t \left(1 - \cos 2t\right)\right]$ 1 m

$\frac{dy}{dx} = \frac{b \left[2 \sin 2t \cos 2t - 2 \sin 2t \left(1 - \cos 2t\right)\right]}{a \left[\sin t \left(-2 \sin 2t\right) + \left(1 + \cos 2t\right)\left(2 \cos 2t\right)\right]}$ 1 m

$= \frac{4 b \cos 3t \sin t}{4 a \cos 3t \cos t} = \frac{b}{a} \tan t = \frac{b}{2} \times 1 = \frac{b}{a}$ 1 m

SECTION - C

20. $f(x) = \sin^2 x - \cos x$

$f'(x) = \sin x\, (2 \cos x + 1)$ 1 m

$f'(x) = 0 \Rightarrow \sin x = 0$ and $2 \cos x + 1 = 0 \Rightarrow x = 0,\ 2\frac{\pi}{3}, \pi$ 2½ m

$f(0) = -1,\ f\left(\frac{2\pi}{3}\right) = \frac{5}{4},\ f(\pi) = 1$ 1½ m

Absolute maximum value is $\frac{5}{4}$ ½ m

Absolute minimum value is – 1 ½ m

21. Two lines $\vec{r} = \vec{a_1} + \lambda\, \vec{b_1}$ and $r = \vec{a_2} + \mu\, \vec{b_2}$ are coplanar

if $\left(\vec{a_2} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right) = 0$ 1 m

Here $\left(-\hat{i} + 3\hat{j} + \hat{k}\right) \cdot \left[\left(\hat{i} - \hat{j} + \hat{k}\right) \times \left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right] = 0$ 2 m

Equation of plane is

$$\left(\vec{r} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right) = 0$$ 1 m

$$\left[\vec{r} - \left(\hat{i} + \hat{j} + \hat{k}\right)\right] \cdot \left[\left(\hat{i} - \hat{j} + \hat{k}\right) \times \left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right] = 0$$

$$\vec{r} \cdot \left(-2\hat{i} - \hat{j} + \hat{k}\right) + 2 = 0$$ 2 m

22. (i) Let (e, e') be the identity element in A

$(a, b) * (e, e') = (a, b) = (e, e') * (a, b)$

$(a e, b + a e') = (a, b)$

$$\left.\begin{array}{r} ae = a \Rightarrow e = 1 \\ b + a e' = b \Rightarrow e' = 0 \end{array}\right] \Rightarrow \text{identity} : (1, 0)$$ 2 ½ m

(ii) Let (x, y) is inverse of $(a, b) \in A$

$(a, b) * (x, y) = (1, 0) = (x, y) * (a, b)$

$(a x, b + a y) = (1, 0)$

$$\left.\begin{array}{l} ax = 1 \Rightarrow x = \frac{1}{a} \\ b + ay = 0 \Rightarrow y = \frac{-b}{a} \end{array}\right] \Rightarrow \text{inverse of } (a, b) = \left(\frac{1}{a}, \frac{-b}{a}\right)$$ 2 ½ m

Inverse of $(5, 3) = \left(\frac{1}{5}, \frac{-3}{5}\right)$ ½ m

Inverse of $\left(\frac{1}{2}, 4\right) = (2, -8)$ ½ m

OR

One – One : - Case I : when x and y are even

$$f(x) = f(y) \Rightarrow x + 1 = y + 1 \Rightarrow x = y$$

Case II : when x and y are odd

$$f(x) = f(y) \Rightarrow x - 1 = y - 1 \Rightarrow x = y$$

Case III : one of them is even and one of them is odd

$$f(x) \neq f(y) \Rightarrow x + 1 \neq y - 1 \Rightarrow x \neq y$$ 2 ½ m

Onto : Let $y \in W$

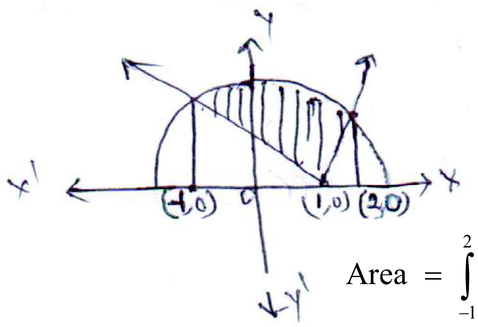
$f(y - 1) = y$ if y is odd

$f(y + 1) = y$ if y is even

So $\forall\ y \in W$, there exist some element in domain of f

$\Rightarrow$ f is invertible 2½ m

$$f^{-1}(x) = \begin{cases} x - 1, & x \text{ is odd} \\ x + 1, & x \text{ is even} \end{cases}$$ 1 m

23.  Figure 1 m

For finding (– 1, 0) , (1, 0) (2, 0) 1½ m

$$\text{Area} = \int_{-1}^{2} \sqrt{5-x^2}\, dx - \int_{-1}^{1} -(x-1)\, dx - \int_{-1}^{2} (x-1)\, dx$$ 1½ m

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} + \left[\frac{(x-1)^2}{2}\right]_{-1}^{1} - \left[\frac{(x-1)^2}{2}\right]_{1}^{2}$$ 1 ½ m

$$= \left(1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right) + \left(1+\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2}\times 4 - \frac{1}{2}\times 1$$

$$= \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2} \text{ sq. units}$$ ½ m

24. $x^2\, dy = (2\,xy + y^2)\, dx$

$$\frac{dy}{dx} = \frac{2\,xy + y^2}{x^2}$$ ½ m

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ 1 m

$$v + x\frac{dv}{dx} = 2v + v^2 \Rightarrow \int \frac{1}{v^2+v}\, dv = \int \frac{1}{x}\, dx$$ 2 m

$$\Rightarrow \log\left|\frac{v}{v+1}\right| = \log x + \log c$$ 1 m

$$\Rightarrow \log\left|\frac{y}{y+x}\right| = \log cx \Rightarrow \frac{y}{y+x} = cx$$ 1 m

$$x = 1,\ y = 1 \Rightarrow c = \frac{1}{2}$$

$x^2 + xy - 2\,y = 0$ ½ m

OR

Given differential equation can be written as

$$\frac{dy}{dx}+\frac{1}{1+x^2}\,y=\frac{e^{m\tan^{-1}x}}{1+x^2}$$ 1 m

Integrating factor is $e^{\tan^{-1}x}$ 1 m

Solution is $y\cdot e^{\tan^{-1}x}=\int\frac{e^{m\tan^{-1}x}}{1+x^2}\cdot e^{\tan^{-1}x}\,dx$ 1½ m

$\Rightarrow\ y\,e^{\tan^{-1}x}=\int e^{(m+1)t}dt$, where $\tan^{-1}x=t$ 1 m

$$=\frac{e^{(m+1)t}}{m+1}=\frac{e^{(m+1)\tan^{-1}x}}{m+1}+c$$ 1 m

$y=1, x=0\ \Rightarrow\ c=\frac{m}{m+1}$

$$y\,e^{\tan^{-1}x}=\frac{e^{(m+1)\tan^{-1}x}}{m+1}+\frac{m}{m+1}$$ ½ m

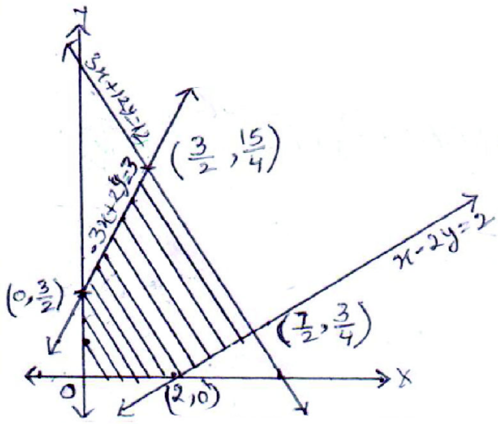
25.

x:	2	3	4	5	6	1 m
P (x):	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$	2 m
x . P (x):	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{12}{15}$	$\frac{20}{15}$	$\frac{30}{15}$	½ m
x^2 P (x):	$\frac{4}{15}$	$\frac{18}{15}$	$\frac{48}{15}$	$\frac{100}{15}$	$\frac{180}{15}$	½ m

Mean $=\sum x\cdot P(x)=\frac{70}{15}=\frac{14}{3}$ 1 m

Variance $=\sum x^2\ P(x)=\ (\text{Mean})^2=\frac{350}{15}-\frac{196}{9}=\frac{14}{9}$ 1 m

26.



Correct graph of three lines 1×3 m

correct shading of feasible region 1 m

vertices are $\left(0, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{15}{4}\right),$

$\left(\frac{7}{2}, \frac{3}{4}\right), (2, 0)$ 1 m

$z = 5x + 2y$ is maximum

at $\left(\frac{7}{2}, \frac{3}{4}\right) = 19$ and

minimum at $\left(0, \frac{3}{2}\right) = 3$ 1 m

QUESTION PAPER CODE 65/3/C

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $d = \left| \frac{\vec{a} \cdot \vec{n} - p}{|\vec{n}|} \right|$, distance $= \frac{13}{7}$ ½ + ½ m

2. $\vec{a} \times \vec{b} = -17\hat{i} + 13\hat{j} + 7\hat{k}$, $|\vec{a} \times \vec{b}| = \sqrt{507}$ ½+½ m

3. $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$, $\theta = \frac{2\pi}{3}$ ½ + ½ m

4. $\frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x}$, Integrating factor $= \log x$ ½ + ½ m

5. $e^{2x} \sin 2x$ 1 m

6. $y = mx, \frac{dy}{dx} = \frac{y}{x}$ ½ + ½ m

SECTION - B

7. $\frac{dx}{at} = a\,[\sin 2t\,(-2\sin 2t) + (1 + \cos 2t)(2\cos 2t)]$ 1 m

$\frac{dy}{dt} = b\,[2\sin 2t\cos 2t - 2\sin 2t\,(1 - \cos 2t)]$ 1 m

$\frac{dy}{dx} = \frac{b\,[2\sin 2t\cos 2t - 2\sin 2t\,(1 - \cos 2t)]}{a\,[\sin t\,(-2\sin 2t) + (1 + \cos 2t)(2\cos 2t)]}$ 1 m

$= \frac{4b\cos 3t\sin t}{4a\cos 3t\cos t} = \frac{b}{a}\tan t = \frac{b}{2} \times 1 = \frac{b}{a}$ 1 m

8. $\int \frac{x+3}{(x+5)^3}\ e^x\ dx$

$\int \frac{1}{(x+5)^2} - \frac{2}{(x+5)^3}\ e^x\ dx$ 1 m

$\int \frac{1}{(x+5)^2}\ e^x\ dx - \int \frac{2}{(x+5)^3}\ e^x\ dx$ ½ m

$= \frac{1}{(x+5)^2}\ e^x + \int \frac{2}{(x+5)^3}\ e^x\ dx - \int \frac{2}{(x+5)^3}\ e^x\ dx$ 2 m

$= \frac{e^x}{(x+5)^2} + c$ ½ m

9. $\begin{matrix} \\ x \\ y \\ z \end{matrix} \begin{matrix} \begin{matrix} F & M & T \end{matrix} \\ \begin{pmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{pmatrix} \end{matrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 5450 \\ 5250 \\ 6625 \end{pmatrix}$ 1½ m

Funds collected by school x : ₹ 5450, school y = ₹ 5250

school z = ₹ 6625 1 m

Total collected funds = ₹ 17325 ½ m

For writing any value 1 m

10. $\overrightarrow{BA} = \hat{i} + (x-1)\hat{j} + 4\hat{k}, \overrightarrow{CA} = \hat{i} - 3\hat{k}, \overrightarrow{DA}\ 3\hat{i} + 3\hat{j} - 2\hat{k}$ 1½ m

$\left[\overrightarrow{BA}, \overrightarrow{CA}, \overrightarrow{DA}\right] = 0$ 1 m

$\begin{vmatrix} 1 & x-1 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$ 1 m

$x = 4$ ½ m

11. $\vec{r} = \left(4\hat{i} + 2\hat{j} + 2\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ $\vec{a} + \lambda\vec{b}$ 1 m

Let L be the foot of perpendicular

Position vector of L is $(2\lambda + 4)\hat{i} + (3\lambda + 2)\hat{j} + (6\lambda + 2)\hat{k}$ ½ m

$\vec{PL} = (2\lambda + 3)\hat{i} + 3\lambda\hat{j} + (6\lambda - 1)\hat{k}$ ½ m

$\overrightarrow{PL} \cdot \vec{b} = 2(2\lambda + 3) + 3(3\lambda) + 6(6\lambda - 1) = 0$

$\Rightarrow \lambda = 0$ 1 m

$\overrightarrow{PL} = 3\hat{i} - \hat{k}$

$\left|\overrightarrow{PL}\right| = \sqrt{10}$ units 1 m

12. $\sin^{-1}(1 - x) - 2\sin^{-1}x = \frac{\pi}{2}$

$(1 - x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$ 1 m

$1 - x = \cos\left(2\sin^{-1}x\right)$ 1 m

$1 - x = 1 - 2x^2$ 1 m

$\Rightarrow x = 0, \frac{1}{2}$ ½ + ½ m

$x = \frac{1}{2}$ is rejected

OR

L.H.S $= 2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$

$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$ 1 m

$$= \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$$ 1 m

$$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right)$$ 1 m

$$= \tan^{-1}\left(\frac{625}{625}\right) = \frac{\pi}{4}$$ 1 m

13. $y = e^{ax} \cos bx$

$y_1 = ae^{ax} \cos bx - b\, e^{ax} \sin b\, x$ 1 m

$y_1 = ay - b\, e^{ax} \sin b\, x$ 1 m

$y_2 = ay_1 - b\, [ae^{ax} \sin bx + b\, e^{ax} \cos b\, x]$ 1 m

$y_2 = ay_1 - a\, be^{ax} \sin bx - b^2\, e^{ax} \cos b\, x$

$y_2 = a\, y_1 - a\, (ay - y_1) - b^2\, y$

$y_2 - 2\, a\, y_1 + (a^2 + b^2)\, y = 0$ 1 m

14. $x^x + x^y + y^x = a^b$

Let $u = x^x$, $v = x^y$, $w = y^x$, $\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ½ m

$\frac{du}{dx} = x^x\,(1 + \log x)$ 1 m

$$\frac{dv}{dx} = x^{y}\left(\frac{y}{x} + \frac{dy}{dx}\log x\right)$$ 1 m

$$\frac{dw}{dx} = y^{x}\left(\frac{x}{y}\cdot\frac{dy}{dx} + \log y\right)$$ 1 m

$$\frac{dy}{dx} = -\left(\frac{x^{x}(1+\log x) + y\,x^{y-1} + y^{x}\log y}{x^{y}\log x + x\,y^{x-1}}\right)$$ ½ m

15. $$A^{2} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$ 1½ m

$$A^{2} - 4A - 51 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix} = O$$ 1 m

$$A^{2} - 4A - 51 = O \Rightarrow A^{-1} = \frac{1}{5}(A - 4I)$$ 1 m

$$A^{-1} = \frac{1}{5}\left\{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$ ½ m

OR

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1 m

Using elementary row operations to reach at

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$ 2 m

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ 1 m

16. $$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 3x+7 & x+6 & x-1 \\ 3x+7 & x-1 & x+2 \\ 3x+7 & x+2 & x+6 \end{vmatrix} = 0$$ 1 m

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 3x+7 & x+6 & x-1 \\ 1 & -7 & 3 \\ 1 & -4 & 7 \end{vmatrix} = 0$$ 2 m

$(3x+7)(-37) = 0 \Rightarrow x = \frac{-7}{3}$ 1 m

17. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx \Rightarrow 2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx$ 1 m

$$2I = \int_0^{\pi/2} \frac{\sec^2 x/2}{2\tan\frac{x}{2} + 1 - \tan^2 x/2} dx$$

$I = -\int_0^1 \frac{1}{(t-1)^2 - (\sqrt{2})^2} dt$, where $\tan\frac{x}{2} = t$ 1½ m

$$I = \left[-\frac{1}{2\sqrt{2}} \log\left|\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right| \right]_0^1$$ 1 m

$$I = \frac{1}{2\sqrt{2}} \log \left| \frac{1+\sqrt{2}}{1-\sqrt{2}} \right|$$ ½ m

OR

$$\int_{-1}^{2} \left(e^{3x} + 7x - 5\right) dx \quad \text{here } h = \frac{3}{n}$$ ½ m

$$= \lim_{h \to 0} h \left[f(-1) + f(-1+h) + \ldots\ldots\ldots.. \right]$$

$$= \lim_{h \to 0} h \left[\left(e^{-3} - 12\right) + \left(e^{-3+3h} + 7h - 12\right) + \ldots\ldots + \left(e^{-3+\overline{n-1}\,h} + 7(n-1)h - 12\right) \right]$$ 1 m

$$= \lim_{h \to 0} h \left[e^{-3}\left(1 + e^{3h} + e^{6h} + \ldots\ldots + e^{3(n-1)h}\right) + 7h\left(1 + 2 + 3 + \ldots.\overline{n-1}\right) - 12\,nh \right]$$ 1 m

$$= \lim_{h \to 0} h \left[\frac{e^{-3}\left(e^{3nh} - 1\right)h}{e^{3h} - 1} + \frac{7(nh)(nh - h)}{2} - 12nh \right]$$ 1 m

$$= \frac{e^{-3}\left(e^{9} - 1\right)}{3} + \frac{63}{2} - 36 = \frac{e^{9} - 1}{3\,e^{3}} - \frac{9}{2}$$ ½ m

18. $\displaystyle\int \frac{x^2}{x^4 + x^2 - 2}\, dx$

$$\int \frac{x^2}{x^4 + x^2 - 2} = \frac{t}{t^2 + t - 2} = \frac{t}{(t+2)(t-1)} \quad \text{where } x^2 = t$$ 1½ m

$$= \frac{2}{3(t+2)} + \frac{1}{3(t-1)}$$ 1½ m

$$\int \frac{x^2}{x^4 + x^2 - 2}\, dx = \int \frac{2}{3\left(x^2 + 2\right)}\, dx + \int \frac{1}{3\left(x^2 - 1\right)}\, dx$$

$$= \frac{2}{3\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{1}{6} \log \left| \frac{x-1}{x+1} \right| + c$$ 1 m

19. Let E_1 : two headed coin is chosen

E_2 : unbiased coin is chosen

A : All 5 tosses are heads ½ m

$$P(E_1) = \frac{1}{5},\ P(E_2) = \frac{4}{5},\ P\left(A/E_1\right) = 1,\ P\left(A/E_2\right) = \frac{1}{32}$$ 2 m

$$P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)}$$ ½ m

$$P\left(E_1/A\right) = \frac{\frac{1}{5}\times 1}{\frac{1}{5}\times 1+\frac{4}{5}\cdot\frac{1}{32}} = \frac{8}{9}$$ 1 m

OR

Let the coin is tossed n times

$$1 - P(0) > \frac{80}{100}$$ 1½ m

$$P(0) < \frac{1}{5}$$ ½ m

$${}^{n}C_1\left(\frac{1}{2}\right)^{n}\left(\frac{1}{2}\right)^{0} < \frac{1}{5}$$ 1 m

$$\left(\frac{1}{2}\right)^{n} < \frac{1}{5} \Rightarrow n \geq 3$$ 1 m

SECTION - C

20. $x^2\,dy = (2\,xy + y^2)\,dx$

$$\frac{dy}{dx} = \frac{2\,xy + y^2}{x^2}$$ ½ m

$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$v + x\frac{dv}{dx} = 2v + v^2 \Rightarrow \int \frac{1}{v^2 + v} dv = \int \frac{1}{x} dx$ 2 m

$\Rightarrow \log \left| \frac{v}{v+1} \right| = \log x + \log c$ 1 m

$\Rightarrow \log \left| \frac{y}{y+x} \right| = \log cx \Rightarrow \frac{y}{y+x} = cx$ 1 m

$x = 1, y = 1 \Rightarrow c = \frac{1}{2}$

$x^2 + xy - 2y = 0$ ½ m

OR

Given differential equation can be written as

$\frac{dy}{dx} + \frac{1}{1+x^2} y = \frac{e^{m \tan^{-1} x}}{1+x^2}$ 1 m

Integrating factor is $e^{\tan^{-1} x}$ 1 m

Solution is $y \cdot e^{\tan^{-1} x} = \int \frac{e^{m \tan^{-1} x}}{1+x^2} \cdot e^{\tan^{-1} x} dx$ 1½ m

$\Rightarrow y e^{\tan^{-1} x} = \int e^{(m+1)t} dt$, where $\tan^{-1} x = t$ 1 m

$= \frac{e^{(m+1)t}}{m+1} = \frac{e^{(m+1)\tan^{-1} x}}{m+1} + c$ 1 m

$y = 1, x = 0 \Rightarrow c = \frac{m}{m+1}$

$y e^{\tan^{-1} x} = \frac{e^{(m+1)\tan^{-1} x}}{m+1} + \frac{m}{m+1}$ ½ m

21. $f(x) = \sin^2 x - \cos x$

$f'(x) = \sin x\,(2\cos x + 1)$ 1 m

$f'(x) = 0 \Rightarrow \sin x = 0$ and $2\cos x + 1 = 0 \Rightarrow x = 0,\ 2\dfrac{\pi}{3},\ \pi$ 2½ m

$f(0) = -1,\ f\left(\dfrac{2\pi}{3}\right) = \dfrac{5}{4},\ f(\pi) = 1$ 1½ m

Absolute maximum value is $\dfrac{5}{4}$ ½ m

Absolute minimum value is – 1 ½ m

22.

x :	2	3	4	5	6	1 m
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$	2 m
x . P (x) :	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{12}{15}$	$\frac{20}{15}$	$\frac{30}{15}$	½ m
x^2 P (x) :	$\frac{4}{15}$	$\frac{18}{15}$	$\frac{48}{15}$	$\frac{100}{15}$	$\frac{180}{15}$	½ m

Mean $= \sum x \cdot P(x) = \dfrac{70}{15} = \dfrac{14}{3}$ 1 m

Variance $= \sum x^2\ P(x) = (\text{Mean})^2 = \dfrac{350}{15} - \dfrac{196}{9} = \dfrac{14}{9}$ 1 m

23. Two lines $\vec{r} = \vec{a}_1 + \lambda\,\vec{b}_1$ and $r = \vec{a}_2 + \mu\,\vec{b}_2$ are coplanar

if $\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_1 \times \vec{b}_2\right) = 0$ 1 m

Here $\left(-\hat{i} + 3\hat{j} + \hat{k}\right)\cdot\left[\left(\hat{i} - \hat{j} + \hat{k}\right)\times\left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right] = 0$ 2 m

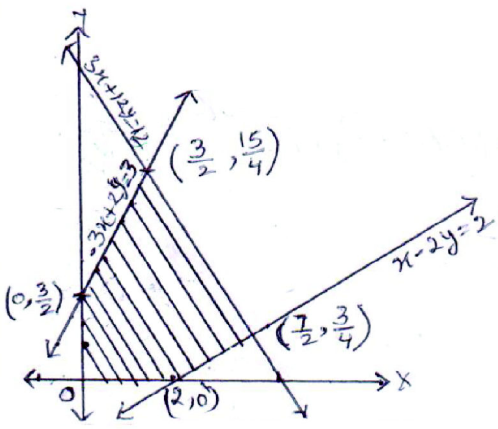
Equation of plane is

$$\left(\vec{r} - \vec{a}_1\right)\cdot\left(\vec{b}_1 \times \vec{b}_2\right) = 0$$ 1 m

$$\left[\vec{r} - \left(\hat{i} + \hat{j} + \hat{k}\right)\right] \cdot \left[\left(\hat{i} - \hat{j} + \hat{k}\right) \times \left(2\,\hat{i} - \hat{j} + 3\,\hat{k}\right)\right] = 0$$

$$\vec{r} \cdot \left(-2\hat{i} - \hat{j} + \hat{k}\right) + 2 = 0$$ 2 m

24.



Correct graph of three lines 1×3 m

correct shading of feasible region 1 m

vertices are $\left(0, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{15}{4}\right),$

$\left(\frac{7}{2}, \frac{3}{4}\right), (2, 0)$ 1 m

$z = 5x + 2y$ is maximum

at $\left(\frac{7}{2}, \frac{3}{4}\right) = 19$ and

minimum at $\left(0, \frac{3}{2}\right) = 3$ 1 m

25. (i) Let (e, e') be the identity element in A

$(a, b) * (e, e') = (a, b) = (e, e') * (a, b)$

$(a\,e, b + a\,e') = (a, b)$

$$\left.\begin{array}{r} ae = a \Rightarrow e = 1 \\ b + a e' = b \Rightarrow e' = 0 \end{array}\right] \Rightarrow \text{identity} : (1, 0)$$ 2 ½ m

(ii) Let (x, y) is inverse of $(a, b) \in A$

$(a, b) * (x, y) = (1, 0) = (x, y) * (a, b)$

$(a\,x, b + a\,y) = (1, 0)$

$$\left.\begin{aligned} ax = 1 \Rightarrow x = \frac{1}{a} \\ b + ay = 0 \Rightarrow y = \frac{-b}{a} \end{aligned}\right] \Rightarrow \text{inverse of } (a, b) = \left(\frac{1}{a}, \frac{-b}{a}\right)$$ 2 ½ m

Inverse of $(5, 3) = \left(\frac{1}{5}, \frac{-3}{5}\right)$ ½ m

Inverse of $\left(\frac{1}{2}, 4\right) = (2, -8)$ ½ m

OR

One – One : - Case I : when x and y are even

$$f(x) = f(y) \Rightarrow x + 1 = y + 1 \Rightarrow x = y$$

Case II : when x and y are odd

$$f(x) = f(y) \Rightarrow x - 1 = y - 1 \Rightarrow x = y$$

Case III : one of them is even and one of them is odd

$$f(x) \neq f(y) \Rightarrow x + 1 \neq y - 1 \Rightarrow x \neq y$$ 2 ½ m

Onto : Let $y \in W$

$f(y - 1) = y$ if y is odd

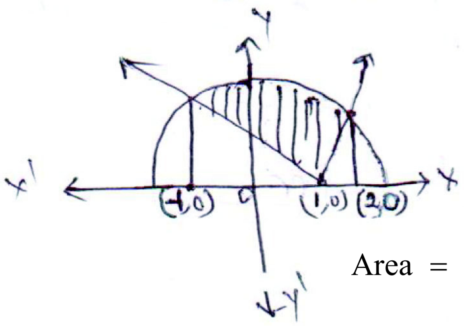
$f(y + 1) = y$ if y is even

So $\forall\ y \in W$, there exist some element in domain of f

$\Rightarrow$ f is invertible 2½ m

$$f^{-1}(x) = \begin{cases} x - 1, & x \text{ is odd} \\ x + 1, & x \text{ is even} \end{cases}$$ 1 m

26. Figure 1 m



For finding (– 1, 0) , (1, 0) (2, 0) 1½ m

$$\text{Area} = \int_{-1}^{2} \sqrt{5-x^2}\, dx - \int_{-1}^{1} -(x-1)\, dx - \int_{-1}^{2} (x-1)\, dx$$ 1½ m

$$= \left[\frac{x}{2} \sqrt{5-x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_{-1}^{2} + \left[\frac{(x-1)^2}{2} \right]_{-1}^{1} - \left[\frac{(x-1)^2}{2} \right]_{1}^{2}$$ 1 ½ m

$$= \left(1 + \frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}}\right) + \left(1 + \frac{5}{2} \sin^{-1} \frac{1}{\sqrt{5}}\right) - \frac{1}{2} \times 4 - \frac{1}{2} \times 1$$

$$= \frac{5}{2} \left(\sin^{-1} \frac{2}{\sqrt{5}} + \sin^{-1} \frac{1}{\sqrt{5}} \right) - \frac{1}{2} \text{ sq. units}$$ ½ m