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Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65/1/G, 65/2/G, 65/3/G

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/G

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$ ½ m

$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$ ½ m

2. $y = e^{-x} + ax + b \Rightarrow y' = -e^{-x} + a$ ½ m

$y'' = e^{-x}$ or $\frac{d^2y}{dx^2} = e^{-x}$ ½ m

3. Order = 2, degree = 2 (any one correct) ½ m

Sum = 2 + 2 = 4 ½ m

4. d.r′s of $\vec{AB}$: 1,–5 – a, b–3 ; d.r′s of $\vec{BC}$ are – 4,16,9–b or d.r′s of $\vec{AC}$:– 3,11 – a, 6 ½ m

getting a = – 1, b = 1, a + b = 0 ½ m

5. $|\vec{a}|\,|\vec{b}|\sin\theta = 16 \Rightarrow \sin\theta = \frac{16}{20} = \frac{4}{5} \Rightarrow \cos\theta = \pm\frac{3}{5}$ ½ m

$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta = \pm 12$ ½ m

6. $d = \frac{|9-6|}{\sqrt{2^2 + (-1)^2 + (-2)^2}}$ ½ m

$= 1$ ½ m

SECTION - B

7. LHS $= \sin\left[\cot^{-1}\left(\frac{2x}{1-x^2}\right) + 2\tan^{-1}x\right]$ 1 m

$= \sin\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2x}{1-x^2}\right) + 2\tan^{-1}x\right]$ 1 m

$= \sin\left[\frac{\pi}{2} - 2\tan^{-1}x + 2\tan^{-1}x\right]$ 1 m

$= \sin\frac{\pi}{2} = 1 =$ R.H.S 1 m

OR

$\tan^{-1}\left(\frac{\frac{x-5}{x-6}+\frac{x+5}{x+6}}{1-\frac{x-5}{x-6}\cdot\frac{x+5}{x+6}}\right) = \frac{\pi}{4}$ 2 m

$\Rightarrow \frac{(x-5)(x+6)+(x+5)(x-6)}{x^2-36-x^2+25} = \tan\frac{\pi}{4}$ 1 m

$\Rightarrow 2x^2 = 49$ ½ m

$\Rightarrow x = \pm\frac{7}{\sqrt{2}}$ ½ m

8. L.H.S. $= \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

$R_1 \to R_1 + b\cdot R_3,\quad R_2 \to R_2 - a\,R_3$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -b\left(1+a^2+b^2\right) \\ 0 & 1+a^2+b^2 & a\left(1+a^2+b^2\right) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ 1+1 m

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ 1 m

Expanding and getting

$$\Delta = \left(1+a^2+b^2\right)^3 = \text{R.H.S.}$$ 1 m

9. $$A^2 = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}$$ 1½ m

$$A^2 - 5A + 4I = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - \begin{pmatrix} 10 & -5 & 5 \\ -5 & 10 & -5 \\ 5 & -5 & 10 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$= O$$ 1 m

Pre multiplying by A^{-1} and getting $A^{-1} = \frac{1}{4}(5I - A)$ ½ m

and $$A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$ 1 m

OR

$$A = IA$$ 1 m

$$\Rightarrow \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$R_3 \rightarrow R_3 - 3R_1$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -9 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix} A$$

$R_1 \rightarrow R_1 - 2R_2 , \quad R_3 \rightarrow R_3 + 5R_2$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{pmatrix} A$$

$R_1 \rightarrow R_1 + R_3 , \quad R_2 \rightarrow R_2 - 2R_3$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{pmatrix} A$$ [operating Row wise to reach at this step] 2½ m

$$A^{-1} = \begin{pmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{pmatrix}$$ ½ m

10. A Candidate who has made an attempt to solve the question to be given 4 marks 4 m

11. $y = -x^3 \log x$ ½ m

$$\frac{dy}{dx} = -\, x^2 (1 + 3 \log x)$$ 1 m

$$\frac{d^2y}{dx^2} = -\ (5x + 6x \log x)$$ 1 m

L.H.S. $= x\,[-(5x + 6x \log x)] + 2x^2 (1 + 3 \log x) + 3x^2$ 1 m

$= 0$ ½ m

$=$ R.H.S.

OR

$f(x) = (x-4)]\,(x-6)\,(x-8)$

$= x^3 - 18\,x^2 + 104\,x - 192$

Being a polynomial function f(x) is continuous

in [4, 10] and differentiable in (4, 10) with

$f'(x) = 3x^2 - 36\,x + 104$ 1+1 m

$\exists\ c \in (4,10) \cdot$ such that $\quad f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$\Rightarrow \quad 3\,c^2 - 36c + 104 = 8$ 1½ m

$\Rightarrow \quad c = 4, 8 \qquad ;\ c = 4 \ \notin \ (4, 10)$

$\therefore \quad c = 8$: verifies the theoren ½ m

12. Given $\dfrac{x}{x-y} = \log a - \log(x - y)$ ½ m

Differentiating both sides and getting $\quad [\because\ x \neq y]$

$$x - 2y + y\,\frac{dy}{dx} = 0$$ 2½ m

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2y - x}{y}$$

$$\Rightarrow \quad \frac{dy}{dx} = 2 - \frac{x}{y}$$ 1 m

13. $I = \int \frac{dx}{x^3 \left(x^5+1\right)^{3/5}}$

$= \int \frac{dx}{x^3 \cdot x^3 \left(1+\frac{1}{x^5}\right)^{3/5}}$ 1½ m

Put $1+\frac{1}{x^5} = t$

$\Rightarrow \frac{dx}{x^6} = -\frac{dt}{5}$ 1 m

$\therefore I = -\frac{1}{5} \int t^{-3/5} dt = -\frac{1}{2} t^{2/5} + C$ 1 m

$= -\frac{1}{2}\left(1+\frac{1}{x^5}\right)^{2/5} + C$ ½ m

14. $I = \int_2^4 |x-2|\,dx + \int_2^4 |x-3|\,dx + \int_2^4 |x-4|\,dx$ ½ m

$= \int_2^4 (x-2)\,dx + \int_2^3 -(x-3)\,dx + \int_3^4 (x-3)\,dx + \int_2^4 -(x-4)\,dx$ 2 m

$= \left[\frac{x^2}{2}-2x\right]_2^4 - \left[\frac{x^2}{2}-3x\right]_2^3 + \left[\frac{x^2}{2}-3x\right]_3^4 - \left[\frac{x^2}{2}-4x\right]_2^4$ 1 m

$= 5$ ½ m

OR

$I = \int_0^{\pi/4} \frac{\sec x}{1+2\sin^2 x}\,dx = \int_0^{\pi/4} \frac{\cos x}{\cos^2 x\left(1+2\sin^2 x\right)}\,dx$

$= \int_0^{\pi/4} \frac{\cos x}{\left(1-\sin x\right)\left(1+\sin x\right)\left(1+2\sin^2 x\right)}\,dx$ 1 m

Put $\sin x = t \Rightarrow \cos x\, dx = dt$, when $x = 0, t = 0$

$$x = \frac{\pi}{4},\ t = \frac{1}{\sqrt{2}}$$ ½ m

$$\therefore \quad I = \int_0^{1/\sqrt{2}} \frac{dt}{(1-t)\,(1+t)\,(1+2t^2)}$$

$$\therefore \quad I = \int_0^{1/\sqrt{2}} \frac{dt}{6(1-t)} + \int_0^{1/\sqrt{2}} \frac{dt}{6(1+t)} + \int_0^{1/\sqrt{2}} \frac{2\ dt}{3(1+2t^2)}$$ 1 m

$$= \left[\frac{1}{6}\log\left|\frac{1+t}{1-t}\right| + \frac{\sqrt{2}}{3}\tan^{-1}\left(\sqrt{2}\ t\right)\right]_0^{\frac{1}{\sqrt{2}}}$$ 1 m

$$= \frac{1}{6}\log\left|\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}\right| + \frac{\sqrt{2}}{3}\tan^{-1}(1)$$

$$= \frac{1}{3}\log\left|\sqrt{2}+1\right| + \frac{\pi}{6\sqrt{2}} \quad or \quad \frac{1}{6}\log\left(3+2\sqrt{2}\right) + \frac{\pi}{6\sqrt{2}}$$ ½ m

15. $$I = \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$$

$$= \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-2\sin\cos x}{2\sin^2 x}\right)dx$$

$$= \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1}{2}\ \text{cosec}^2 x - \cot x\right)dx$$ 1½ m

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

when $x = \frac{\pi}{4},\ t = \frac{\pi}{2};\ x = \frac{\pi}{2},\ t = \pi$ 1 m

$$\therefore\ I = \frac{1}{2}\int_{\pi/2}^{\pi} e^t\left(\frac{1}{2}\operatorname{cosec}^2\frac{t}{2} - \cot\frac{t}{2}\right)dt$$

$$= -\frac{1}{2}\left[\cot\frac{t}{2}\cdot e^t\right]_{\frac{\pi}{2}}^{\pi}$$ 1 m

$$= \frac{e^{\pi/2}}{2}$$ ½ m

16. Let $\overrightarrow{OA} = 4\hat{i} + 8\hat{j} + 12\hat{k},\ \overrightarrow{OB} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

$\overrightarrow{OC} = 3\hat{i} + 5\hat{j} + 4\hat{k},\ \overrightarrow{OD} = 5\hat{i} + 8\hat{j} + 5\hat{k}$

$\overrightarrow{AB} = -2\hat{i} - 4\hat{j} - 6\hat{k},\ \overrightarrow{AC} = -\hat{i} - 3\hat{j} - 8\hat{k},\ \overrightarrow{AD} = \hat{i} - 7\hat{k}$ 1½ m

Now, $\left[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}\right] = \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & -7 \end{vmatrix} = 0$ 2 m

$\therefore$ A, B, C, D are coplanar ½ m

17. Let E_1 : Event that transferred ball is black

E_2 : Event that transferred ball is Red

E_3 : Event that balls drawn are black

$P(E_1) = \frac{5}{9},\quad P(E_2) = \frac{4}{9}$ 1 m

$P(A/E_1) = \frac{^5C_2}{^8C_2} = \frac{5}{14},\quad P(A/E_2) = \frac{^4C_2}{^8C_2} = \frac{3}{14}$ 1 m

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2)}$$ ½ m

$$= \frac{\frac{5}{9} \times \frac{5}{14}}{\frac{5}{9} \times \frac{5}{14} + \frac{4}{9} \times \frac{3}{14}}$$ 1 m

$$= \frac{25}{37}$$ ½ m

18. Equation of line joining $(4, 3, 2)$ and $(1, -1, 0)$ is

$$\frac{x-4}{-3} = \frac{y-3}{-4} = \frac{z-2}{-2}$$ ½ m

Equation of line joining $(1, 2, -1)$ and $(2, 1, 1)$ is

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$$ ½ m

Let equation of the required line be

$$\frac{x-1}{a} = \frac{y+1}{b} = \frac{z-1}{c} = \lambda \quad \text{................ (i)}$$ ½ m

According to the question $3a + 4b + 2c = 0$

$$a - b + 2c = 0$$ 1 m

Solving, $\frac{a}{10} = \frac{b}{-4} = \frac{c}{-7} = \mu$

$\Rightarrow \quad a = 10\mu, \; b = -4\mu, \; c = -7\mu$ ½ m.

(i) $\Rightarrow$ Equation of the line is

$$\frac{x-1}{10} = \frac{y+1}{-4} = \frac{z-1}{-7}$$ [cartesian form] ½ m

Vector form, $\vec{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 7\hat{k}\right)$ ½ m

19. $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} H & R & HW \end{array} \\ \begin{bmatrix} 3 & 4 & 6 \\ 4 & 5 & 3 \\ 6 & 2 & 4 \end{bmatrix} \end{array} \begin{bmatrix} 2500 \\ 3100 \\ 5100 \end{bmatrix}$ 1 m

$$= \begin{bmatrix} 50500 \\ 40800 \\ 41600 \end{bmatrix}$$ 1 m

Hence money awarded by A = Rs. 50500

money awarded by B = Rs. 40800 1 m

money awarded by C = Rs. 41600

Respect for elders or Any relevant value 1 m

SECTION - C

20. $(a, b) * (c, d) = (a + c, b + d) \quad \forall \ a, b, c, d \in R$

Since $a + c \in R$ and $b + d \in R \Rightarrow (a + c, b + d) \in R \times R$ 1½ m

i.e. '*' is binary operation

For commutative

consider $(c, d) * (a, b) = (c + a, \ d + b)$

$= (a + c, \ b + d)$

$= (a, b) * \ (c, d)$ 1½ m

$\Rightarrow$ '*' is commutative

For Associative

Let $(a, b), \ (c, d), \ (e, f) \in R \times R = A$

$[(a, b) * (c, d)] * (e, f) = (a + c, \ b + d) * (e, f)$

$= (a + c + e, b + d + f)$ ....(i)

again $(a, b) * [(c, d)] * (e, f)] = (a, b) * (c + e, d + f)$

$= (a + c + e, b + d + f)$ ....(ii) 1½ m

(i) & (ii) $\Rightarrow$ '*' is associative

For identity element

Let $(e_1, e_2) \in R \times R$ be the identity element (if exists)

then $(a, b) * (e_1, e_2) = (a, b) = (e_1, e_2) * (a, b)$

$\Rightarrow (e_1, e_2) = (0, 0) \in R \times R$ 1½ m

OR

$f(x) = x^2 - x; \quad x \in \{-1, 0, 1, 2\}$

$f(-1) = 2, \; f(0) = 0, \; f(1) = 0, \; f(2) = 2$

$\therefore \; f = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

$$g(x) = 2\left| x - \frac{1}{2} \right| - 1 \quad \forall \; x \in \{-1, 0, 1, 2\}$$

$g(-1) = 2, \; g(0) = 0, \; g(1) = 0, \; g(2) = 2$

$\therefore \; g = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

$(g\,of)(x) = g(f(-1), \; g(f(0), \; g(f(1), \; g(f(2) \quad \forall \; x \in A$

$= 2, 0, 0, 2$

$\therefore \; g\,of = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

Hence $f = g = g\,o\,f$

21. Given curve cuts the x – axis when y = 0 ½ m

when y = 0, x = 7, hence point is (7, 0) ½ m

$$\frac{dy}{dx} = \frac{1 - y(2x - 5)}{x^2 - 5x + 6}$$ 2½ m

$$\left.\frac{dy}{dx}\right]_{(7,0)} = \frac{1}{20}$$ ½ m

Equation of the tangent is $y - 0 = \dfrac{1}{20}(x - 7)$ 1 m

$\Rightarrow \; x - 20y = 7$

Equation of the normal is $y-0 = -20(x-7)$ 1 m

$$\Rightarrow \quad 20x + y = -7$$

OR

$f(x) = \cos^2 x + \sin x, \quad x \in [0, \pi]$

$f'(x) = \cos x(-2\sin x + 1)$ 1 m

For extremum, $f'(x) = 0 \Rightarrow x = \frac{\pi}{2}$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}$ 1½ m

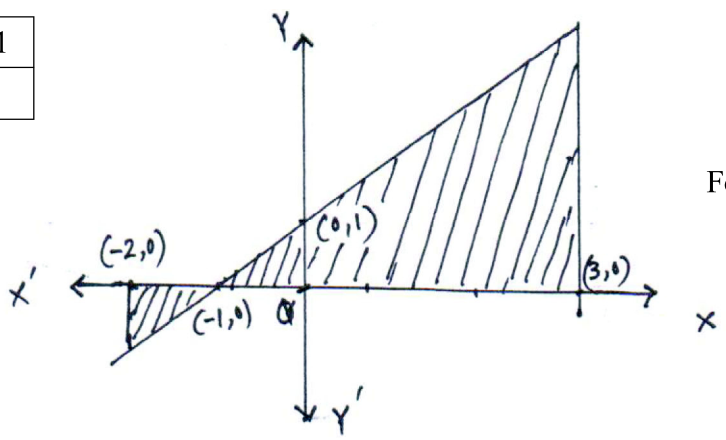
Now $f(0) = 1, \; f\left(\frac{\pi}{6}\right) = \frac{5}{4}, \; f\left(\frac{\pi}{2}\right) = 1, \; f\left(\frac{5\pi}{6}\right) = \frac{5}{4}, \; f(\pi) = 1$ 1½ m

Absolute max. is $\frac{5}{4}$ at $x = \frac{\pi}{2}$ and $\frac{5\pi}{6}$ 1 m

Absolute min. is 1 at $x = 0, \frac{\pi}{6}$ and π 1 m

22. $y = x + 1, \quad x = -2, \quad x = 3$

x	0	–1
y	1	0



For correct figure 1 m

Reqd area $= \left| \int_{-2}^{-1} (x+1)\,dx \right| + \int_{-1}^{3} (x+1)\,dx$ 2 m

$= \left| \left(\frac{x^2}{2} + x \right)_{-2}^{-1} \right| + \left(\frac{x^2}{2} + x \right)_{-1}^{3}$ 2 m

$= \frac{17}{2}$ sq. units 1 m

23. $(y - \sin x)\,dx + (\tan x)\,dy = 0 \Rightarrow \dfrac{dy}{dx} + \cot x\, y = \cos x$ 1 m

Linear diff. equ. with $P = \cot x,\ Q = \cos x$

I.F. $= \sin x$ 1 m

Solution is $y \cdot \sin x = \int \cos x \cdot \sin x\, dx + c$

$= -\dfrac{1}{4}\cos 2x + c$ 2 m

when $x = 0,\ y = 0 \Rightarrow c = \dfrac{1}{4}$ 1 m

Particular solution is

$y \sin x = \dfrac{1}{4}(-\cos 2x + 1) = \dfrac{\sin^2 x}{2}$

$\Rightarrow y = \dfrac{1}{2}\sin x$ 1 m

24. $d \cdot r's$ of first line : $k - 5, 1, 2k + 1$ 1 m

$d \cdot r's$ of 2nd line : $-1, k, 5$ 1 m

$\because$ lines are $\perp$ $\therefore$ $-1(k - 5) + k(1) + 5(2k + 1) = 0$

$\Rightarrow k = -1$ 1 m

Eqns of lines become $\dfrac{x+3}{-6} = \dfrac{y-1}{-1} = \dfrac{z-5}{-1}$ and $\dfrac{x+2}{-1} = \dfrac{y-2}{-1} = \dfrac{z}{5}$ 1 m

Eqn of plane containing these two lines is

$$\begin{vmatrix} x+2 & y-2 & z \\ -6 & 1 & -1 \\ -1 & -1 & 5 \end{vmatrix} = 0$$ 1 m

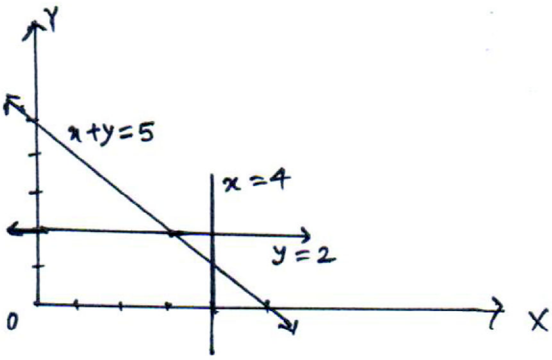
$\Rightarrow 4x + 31y + 7z = 54$ 1 m

25. Let x kg of B_1 and y kg of B_2 is taken

then to minimize $Z = 5x + 8y$ 1 m

subject to the following constraints 3 m

$x + y = 5, \quad x \leq 4, \quad y \geq 2$



Graph 2 m

26. Let x denote no. of heads

here $p = \frac{1}{2}, \quad q = \frac{1}{2}$ 1 m

$$P(x=r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r}$$

$$= {}^nC_r \left(\frac{1}{2}\right)^n$$ 1 m

Now $P(x=1) = {}^nC_1 \left(\frac{1}{2}\right)^n$

$$P(x=2) = {}^nC_2 \left(\frac{1}{2}\right)^n$$ 1½ m

$$P(x=3) = {}^nC_3 \left(\frac{1}{2}\right)^n$$

According to the question

$$2.\ {}^nC_2 \left(\frac{1}{2}\right)^n = \left({}^nC_1 + {}^nC_3\right)\left(\frac{1}{2}\right)^n$$ 2 m

$\Rightarrow \quad n = 2$ or 7 ½ m

n can not be 2 Hence $n = 7$

QUESTION PAPER CODE 65/2/G

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. d.r′s of $\overrightarrow{AB}$: 1,–5 – a, b – 3 ; d.r′s of $\overrightarrow{BC}$ are – 4,16,9 – b or d.r′s of $\overrightarrow{AC}$: – 3,11 – a, 6 ½ m

getting a = – 1, b = 1, a + b = 0 ½ m

2. $|\vec{a}|\,|\vec{b}|\sin\theta = 16 \Rightarrow \sin\theta = \frac{16}{20} = \frac{4}{5} \Rightarrow \cos\theta = \pm\frac{3}{5}$ ½ m

$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta = \pm 12$ ½ m

3. $d = \frac{|9-6|}{\sqrt{2^2+(-1)^2+(-2)^2}}$ ½ m

$= 1$ ½ m

4. $3\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix} - \begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$ ½ m

$= \begin{bmatrix}8 & 7\\ 6 & 2\end{bmatrix}$ ½ m

5. $y = e^{-x} + ax + b \Rightarrow y' = -e^{-x} + a$ ½ m

$y'' = e^{-x}$ or $\frac{d^2y}{dx^2} = e^{-x}$ ½ m

6. Order = 2, degree = 2 (any one correct) ½ m

Sum = 2 + 2 = 4 ½ m

SECTION - B

7. Given $\frac{x}{x-y} = \log a - \log (x - y)$ ½ m

Differentiating both sides and getting [$\because$ $x \neq y$]

$$x - 2y + y\frac{dy}{dx} = 0$$ 2½ m

$$\Rightarrow \frac{dy}{dx} = \frac{2y - x}{y}$$

$$\Rightarrow \frac{dy}{dx} = 2 - \frac{x}{y}$$ 1 m

8. $I = \int \frac{dx}{x^3 \left(x^5 + 1\right)^{3/5}}$

$$= \int \frac{dx}{x^3 \cdot x^3 \left(1 + \frac{1}{x^5}\right)^{3/5}}$$ 1½ m

Put $1 + \frac{1}{x^5} = t$

$$\Rightarrow \frac{dx}{x^6} = -\frac{dt}{5}$$ 1 m

$$\therefore \quad I = -\frac{1}{5}\int t^{-3/5} dt = -\frac{1}{2} t^{2/5} + C$$ 1 m

$$= -\frac{1}{2}\left(1 + \frac{1}{x^5}\right)^{2/5} + C$$ ½ m

9. $\quad I = \int_2^4 |x-2|\,dx + \int_2^4 |x-3|\,dx + \int_2^4 |x-4|\,dx$ ½ m

$$= \int_2^4 (x-2)\,dx + \int_2^3 -(x-3)\,dx + \int_3^4 (x-3)\,dx + \int_2^4 -(x-4)\,dx$$ 2 m

$$= \left[\frac{x^2}{2} - 2x\right]_2^4 - \left[\frac{x^2}{2} - 3x\right]_2^3 + \left[\frac{x^2}{2} - 3x\right]_3^4 - \left[\frac{x^2}{2} - 4x\right]_2^4$$ 1 m

$= 5$ ½ m

OR

$$I = \int_0^{\pi/4} \frac{\sec x}{1 + 2\sin^2 x}\,dx = \int_0^{\pi/4} \frac{\cos x}{\cos^2 x\,(1 + 2\sin^2 x)}\,dx$$

$$= \int_0^{\pi/4} \frac{\cos x}{(1 - \sin x)(1 + \sin x)(1 + 2\sin^2 x)}\,dx$$ 1 m

Put $\sin x = t \Rightarrow \cos x\,dx = dt$, when $x = 0, t = 0$

$x = \frac{\pi}{4},\ t = \frac{1}{\sqrt{2}}$ ½ m

$$\therefore \quad I = \int_0^{1/\sqrt{2}} \frac{dt}{(1-t)\,(1+t)\,(1+2t^2)}$$

$$\therefore \quad I = \int_0^{1/\sqrt{2}} \frac{dt}{6(1-t)} + \int_0^{1/\sqrt{2}} \frac{dt}{6(1+t)} + \int_0^{1/\sqrt{2}} \frac{2\,dt}{3(1+2t^2)}$$ 1 m

$$= \left[\frac{1}{6}\log\left|\frac{1+t}{1-t}\right| + \frac{\sqrt{2}}{3}\tan^{-1}\left(\sqrt{2}\,t\right)\right]_0^{\frac{1}{\sqrt{2}}}$$ 1 m

$$= \frac{1}{6}\log\left|\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}\right| + \frac{\sqrt{2}}{3}\tan^{-1}(1)$$

$$= \frac{1}{3} \log \left| \sqrt{2} + 1 \right| + \frac{\pi}{6\sqrt{2}} \quad or \quad \frac{1}{6} \log \left(3 + 2\sqrt{2}\right) + \frac{\pi}{6\sqrt{2}}$$ ½ m

10. $$\text{LHS} = \sin\left[\cot^{-1}\left(\frac{2x}{1-x^2}\right) + 2\tan^{-1}x\right]$$ 1 m

$$= \sin\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2x}{1-x^2}\right) + 2\tan^{-1}x\right]$$ 1 m

$$= \sin\left[\frac{\pi}{2} - 2\tan^{-1}x + 2\tan^{-1}x\right]$$ 1 m

$$= \sin\frac{\pi}{2} = 1 = \text{R.H.S}$$ 1 m

OR

$$\tan^{-1}\left(\frac{\frac{x-5}{x-6} + \frac{x+5}{x+6}}{1 - \frac{x-5}{x-6}\cdot\frac{x+5}{x+6}}\right) = \frac{\pi}{4}$$ 2 m

$$\Rightarrow \quad \frac{(x-5)(x+6) + (x+5)(x-6)}{x^2 - 36 - x^2 + 25} = \tan\frac{\pi}{4}$$ 1 m

$$\Rightarrow \quad 2x^2 = 49$$ ½ m

$$\Rightarrow \quad x = \pm\frac{7}{\sqrt{2}}$$ ½ m

11. $$\text{L.H.S.} = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$R_1 \to R_1 + b\cdot R_3, \quad R_2 \to R_2 - a\,R_3$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -b\left(1+a^2+b^2\right) \\ 0 & 1+a^2+b^2 & a\left(1+a^2+b^2\right) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ 1+1 m

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ 1 m

Expanding and getting

$$\Delta = \left(1+a^2+b^2\right)^3 = \text{R.H.S.}$$ 1 m

12. $$A^2 = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}$$ 1½ m

$$A^2 - 5A + 4I = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - \begin{pmatrix} 10 & -5 & 5 \\ -5 & 10 & -5 \\ 5 & -5 & 10 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$= O$$ 1 m

Pre multiplying by A^{-1} and getting $A^{-1} = \frac{1}{4}(5I - A)$ ½ m

and $$A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$ 1 m

OR

$$A = IA$$ 1 m

$$\Rightarrow \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\begin{pmatrix}1 & 2 & 3\\0 & 1 & 2\\3 & 1 & 0\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{pmatrix} A$$

$R_3 \to R_3 - 3R_1$

$$\begin{pmatrix}1 & 2 & 3\\0 & 1 & 2\\0 & -5 & -9\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & -3 & 1\end{pmatrix} A$$

$R_1 \to R_1 - 2R_2, \quad R_3 \to R_3 + 5R_2$

$$\begin{pmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}-2 & 1 & 0\\1 & 0 & 0\\5 & -3 & 1\end{pmatrix} A$$

$R_1 \to R_1 + R_3, \quad R_2 \to R_2 - 2R_3$

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}3 & -2 & 1\\-9 & 6 & -2\\5 & -3 & 1\end{pmatrix} A$$ [operating Row wise to reach at this step] 2½ m

$$A^{-1} = \begin{pmatrix}3 & -2 & 1\\-9 & 6 & -2\\5 & -3 & 1\end{pmatrix}$$ ½ m

13. A Candidate who has made an attempt to solve the question to be given 4 marks 4 m

14. $y = -x^3 \log x$ ½ m

$$\frac{dy}{dx} = -x^2(1 + 3\log x)$$ 1 m

$\frac{d^2y}{dx^2} = -(5x + 6x \log x)$ 1 m

L.H.S. $= x[-(5x + 6x \log x)] + 2x^2(1 + 3 \log x) + 3x^2$ 1 m

$= 0$ ½ m

$=$ R.H.S.

OR

$f(x) = (x-4)](x-6)(x-8)$

$= x^3 - 18x^2 + 104x - 192$

Being a polynomial function f (x) is continuous

in [4, 10] and differentiable in (4, 10) with

$f'(x) = 3x^2 - 36x + 104$ 1+1 m

$\exists\ c \in (4,10) \cdot$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

$\Rightarrow \quad 3c^2 - 36c + 104 = 8$ 1½ m

$\Rightarrow \quad c = 4, 8 \quad ; \ c = 4 \notin (4, 10)$

$\therefore \quad c = 8$: verifies the theoren ½ m

15. Equation of line joining (4, 3, 2) and (1, – 1, 0) is

$$\frac{x-4}{-3} = \frac{y-3}{-4} = \frac{z-2}{-2}$$ ½ m

Equation of line joining (1, 2, – 1) and (2, 1, 1) is

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$$ ½ m

Let equation of the required line be

$$\frac{x-1}{a} = \frac{y+1}{b} = \frac{z-1}{c} = \lambda \ .... (i)$$ ½ m

According to the question $3a + 4b + 2c = 0$

$$a - b + 2c = 0$$ 1 m

Solving, $\frac{a}{10} = \frac{b}{-4} = \frac{c}{-7} = \mu$

$\Rightarrow \quad a = 10\mu, \; b = -4\mu, c = -7\mu$ ½ m.

(i) $\Rightarrow$ Equation of the line is

$$\frac{x-1}{10} = \frac{y+1}{-4} = \frac{z-1}{-7} \text{ [cartesian form]}$$ ½ m

Vector form, $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 7\hat{k})$ ½ m

16. 1 m

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} H & R & HW \end{array} \\ \begin{bmatrix} 3 & 4 & 6 \\ 4 & 5 & 3 \\ 6 & 2 & 4 \end{bmatrix} \end{array} \begin{bmatrix} 2500 \\ 3100 \\ 5100 \end{bmatrix}$$

$$= \begin{bmatrix} 50500 \\ 40800 \\ 41600 \end{bmatrix}$$ 1 m

Hence money awarded by A = Rs. 50500

money awarded by B = Rs. 40800 1 m

money awarded by C = Rs. 41600

Respect for elders or Any relevant value 1 m

17. $I = \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

$$= \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-2\sin\cos x}{2\sin^2 x}\right)dx$$

$$= \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1}{2}\ \text{cosec}^2 x - \cot x\right)dx$$ 1½ m

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

when $x = \frac{\pi}{4},\ t = \frac{\pi}{2};\ x = \frac{\pi}{2},\ t = \pi$ 1 m

$\therefore\ I = \frac{1}{2}\int_{\pi/2}^{\pi} e^{t}\left(\frac{1}{2}\ \text{cosec}^2\frac{t}{2} - \cot\frac{t}{2}\right)dt$

$$= -\frac{1}{2}\left[\cot\frac{t}{2}\cdot e^{t}\right]_{\frac{\pi}{2}}^{\pi}$$ 1 m

$$= \frac{e^{\pi/2}}{2}$$ ½ m

18. Let $\overrightarrow{OA} = 4\hat{i}+8\hat{j}+12\hat{k},\ \overrightarrow{OB} = 2\hat{i}+4\hat{j}+6\hat{k}$

$\overrightarrow{OC} = 3\hat{i}+5\hat{j}+4\hat{k},\ \overrightarrow{OD} = 5\hat{i}+8\hat{j}+5\hat{k}$

$\overrightarrow{AB} = -2\hat{i}-4\hat{j}-6\hat{k},\ \overrightarrow{AC} = -\hat{i}-3\hat{j}-8\hat{k},\ \overrightarrow{AD} = \hat{i}-7\hat{k}$ 1½ m

Now, $\left[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}\right] = \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & -7 \end{vmatrix} = 0$ 2 m

$\therefore$ A, B, C, D are coplanar ½ m

19. Let E_1 : Event that transferred ball is black

E_2 : Event that transferred ball is Red

E_3 : Event that balls drawn are black

$P(E_1) = \frac{5}{9}, \quad P(E_2) = \frac{4}{9}$ 1 m

$P(A/E_1) = \frac{{}^5C_2}{{}^8C_2} = \frac{5}{14}, \quad P(A/E_2) = \frac{{}^4C_2}{{}^8C_2} = \frac{3}{14}$ 1 m

$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2)}$ ½ m

$= \frac{\frac{5}{9} \times \frac{5}{14}}{\frac{5}{9} \times \frac{5}{14} + \frac{4}{9} \times \frac{3}{14}}$ 1 m

$= \frac{25}{37}$ ½ m

SECTION - C

20. $(y - \sin x)\, dx + (\tan x)\, dy = 0 \Rightarrow \frac{dy}{dx} + \cot x\, y = \cos x$ 1 m

Linear diff. equ. with $P = \cot x$, $Q = \cos x$

I.F. $= \sin x$ 1 m

Solution is $y \cdot \sin x = \int \cos x \cdot \sin x\, dx + c$

$= -\frac{1}{4} \cos 2x + c$ 2 m

when $x = 0$, $y = 0 \Rightarrow c = \frac{1}{4}$ 1 m

Particular solution is

$y \sin x = \frac{1}{4}(-\cos 2x + 1) = \frac{\sin^2 x}{2}$

$\Rightarrow y = \frac{1}{2} \sin x$ 1 m

21. $d\cdot r's$ of first line : $k-5, 1, 2k+1$ 1 m

$d\cdot r's$ of 2nd line : $-1, k, 5$ 1 m

$\because$ lines are $\perp$ $\therefore$ $-1(k-5)+k(1)+5(2k+1)=0$

$\Rightarrow \quad k=-1$ 1 m

Eqns of lines become $\frac{x+3}{-6}=\frac{y-1}{-1}=\frac{z-5}{-1}$ and $\frac{x+2}{-1}=\frac{y-2}{-1}=\frac{z}{5}$ 1 m

Eqn of plane containing these two lines is

$$\begin{vmatrix} x+2 & y-2 & z \\ -6 & 1 & -1 \\ -1 & -1 & 5 \end{vmatrix} = 0$$ 1 m

$\Rightarrow \quad 4x+31y+7z=54$ 1 m

22. $(a,b) * (c,d) = (a+c, b+d) \quad \forall\ a, b, c, d \in R$

Since $a+c \in R$ and $b+d \in R \Rightarrow (a+c, b+d) \in R \times R$ 1½ m

i.e. '*' is binary operation

For commutative

consider $(c, d) * (a, b) = (c+a,\ d+b)$

$= (a+c,\ b+d)$

$= (a, b) * (c, d)$ 1½ m

$\Rightarrow$ '*' is commutative

For Associative

Let $(a, b), (c, d), (e, f) \in R \times R = A$

$[(a, b) * (c, d)] * (e, f) = (a+c,\ b+d) * (e, f)$

$= (a+c+e, b+d+f)$ ....(i)

again $(a, b) * [(c, d)] * (e, f)] = (a, b) * (c+e, d+f)$

$= (a+c+e, b+d+f)$ ....(ii) 1½ m

(i) & (ii) $\Rightarrow$ '*' is associative

For identity element

Let $(e_1, e_2) \in R \times R$ be the identity element (if exists)

then $(a, b) * (e_1, e_2) = (a, b) = (e_1, e_2) * (a, b)$

$\Rightarrow$ $(e_1, e_2) = (0, 0) \in R \times R$ 1½ m

OR

$f(x) = x^2 - x; \quad x \in \{-1, 0, 1, 2\}$

$f(-1) = 2, \; f(0) = 0, \; f(1) = 0, \; f(2) = 2$

$\therefore \; f = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

$$g(x) = 2\left| x - \frac{1}{2} \right| - 1 \quad \forall \; x \in \{-1, 0, 1, 2\}$$

$g(-1) = 2, \; g(0) = 0, \; g(1) = 0, \; g(2) = 2$

$\therefore \; g = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

$(g of)(x) = g(f(-1), \; g(f(0), \; g(f(1), \; g(f(2) \quad \forall \; x \in A$

$= 2, 0, 0, 2$

$\therefore \; g of = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

Hence $f = g = g o f$

23. Given curve cuts the x – axis when $y = 0$ ½ m

when $y = 0$, $x = 7$, hence point is $(7, 0)$ ½ m

$$\frac{dy}{dx} = \frac{1 - y(2x - 5)}{x^2 - 5x + 6}$$ 2½ m

$$\left.\frac{dy}{dx}\right]_{(7,0)} = \frac{1}{20}$$ ½ m

Equation of the tangent is $y - 0 = \frac{1}{20}(x - 7)$ 1 m

$\Rightarrow$ $x - 20y = 7$

Equation of the normal is $\quad y - 0 = -20(x - 7)$ 1 m

$$\Rightarrow \quad 20x + y = -7$$

OR

$f(x) = \cos^2 x + \sin x, \quad x \in [0, \pi]$

$f'(x) = \cos x(-2\sin x + 1)$ 1 m

For extremum, $f'(x) = 0 \Rightarrow x = \dfrac{\pi}{2}$ or $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ 1½ m

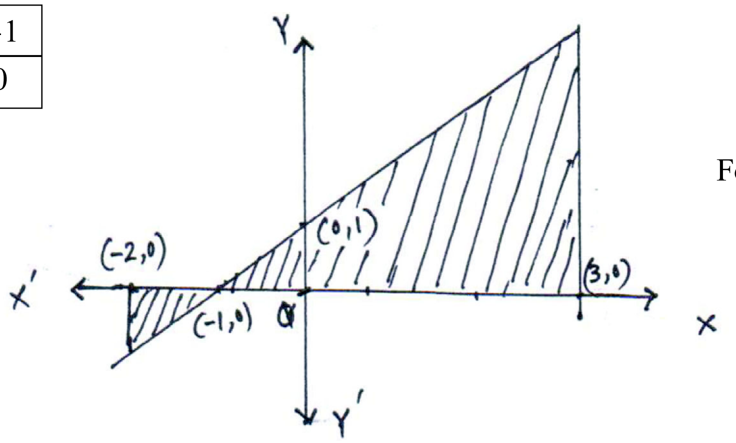
Now $f(0) = 1, \; f\left(\dfrac{\pi}{6}\right) = \dfrac{5}{4}, \; f\left(\dfrac{\pi}{2}\right) = 1, \; f\left(\dfrac{5\pi}{6}\right) = \dfrac{5}{4}, \; f(\pi) = 1$ 1½ m

Absolute max. is $\dfrac{5}{4}$ at $x = \dfrac{\pi}{2}$ and $\dfrac{5\pi}{6}$ 1 m

Absolute min. is 1 at $x = 0, \; \dfrac{\pi}{6}$ and π 1 m

24. $y = x + 1, \quad x = -2, \quad x = 3$

x	0	–1
y	1	0



For correct figure 1 m

$$\text{Reqd area} = \left|\int_{-2}^{-1}(x+1)\,dx\right| + \int_{-1}^{3}(x+1)\,dx$$ 2 m

$$= \left|\left(\frac{x^2}{2} + x\right)_{-2}^{-1}\right| + \left(\frac{x^2}{2} + x\right)_{-1}^{3}$$ 2 m

$$= \frac{17}{2} \text{ sq. units}$$ 1 m

25. Let x denote no. of heads

here $p = \frac{1}{2}, \quad q = \frac{1}{2}$ 1 m

$$P(x = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r}$$

$$= {}^nC_r \left(\frac{1}{2}\right)^n$$ 1 m

Now $P(x = 1) = {}^nC_1 \left(\frac{1}{2}\right)^n$

$$P(x = 2) = {}^nC_2 \left(\frac{1}{2}\right)^n$$ 1½ m

$$P(x = 3) = {}^nC_3 \left(\frac{1}{2}\right)^n$$

According to the question

$$2.\ {}^nC_2 \left(\frac{1}{2}\right)^n = \left({}^nC_1 + {}^nC_3\right)\left(\frac{1}{2}\right)^n$$ 2 m

$\Rightarrow \quad n = 2 \text{ or } 7$ ½ m

n can not be 2 Hence $n = 7$

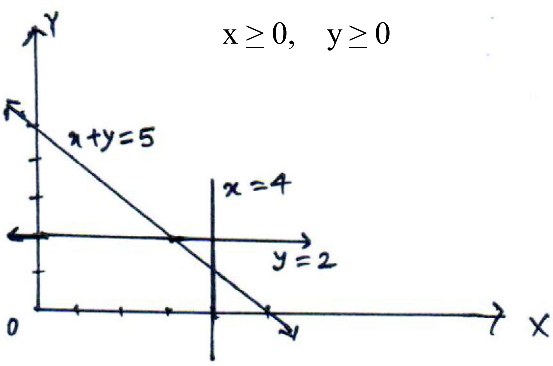
26. Let x kg of B_1 and y kg of B_2 is taken

then to minimize $Z = 5x + 8y$ 1 m

subject to the following constraints 3 m

$x + y = 5, \quad x \leq 4, \quad y \geq 2$

$x \geq 0, \quad y \geq 0$



Graph 2 m

QUESTION PAPER CODE 65/3/G

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. Order = 2, degree = 2 (any one correct) ½ m

Sum = 2 + 2 = 4 ½ m

2. $3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$ ½ m

$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$ ½ m

3. $y = e^{-x} + ax + b \Rightarrow y' = -e^{-x} + a$ ½ m

$y'' = e^{-x}$ or $\frac{d^2y}{dx^2} = e^{-x}$ ½ m

4. $d = \frac{|9-6|}{\sqrt{2^2 + (-1)^2 + (-2)^2}}$ ½ m

$= 1$ ½ m

5. d.r′s of $\vec{AB}$: $1, -5-a, b-3$; d.r′s of $\vec{BC}$ are $-4, 16, 9-b$ or d.r′s of $\vec{AC}$: $-3, 11-a, 6$ ½ m

getting $a = -1$, $b = 1$, $a + b = 0$ ½ m

6. $|\vec{a}|\,|\vec{b}|\sin\theta = 16 \Rightarrow \sin\theta = \frac{16}{20} = \frac{4}{5} \Rightarrow \cos\theta = \pm\frac{3}{5}$ ½ m

$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta = \pm 12$ ½ m

SECTION - B

7. Let E_1 : Event that transferred ball is black

E_2 : Event that transferred ball is Red

E_3 : Event that balls drawn are black

$P(E_1) = \frac{5}{9}, \quad P(E_2) = \frac{4}{9}$ 1 m

$P(A/E_1) = \frac{{}^5C_2}{{}^8C_2} = \frac{5}{14}, \quad P(A/E_2) = \frac{{}^4C_2}{{}^8C_2} = \frac{3}{14}$ 1 m

$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2)}$ ½ m

$= \frac{\frac{5}{9} \times \frac{5}{14}}{\frac{5}{9} \times \frac{5}{14} + \frac{4}{9} \times \frac{3}{14}}$ 1 m

$= \frac{25}{37}$ ½ m

8. Equation of line joining (4, 3, 2) and (1, – 1, 0) is

$$\frac{x-4}{-3} = \frac{y-3}{-4} = \frac{z-2}{-2}$$ ½ m

Equation of line joining (1, 2, – 1) and (2, 1, 1) is

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$$ ½ m

Let equation of the required line be

$$\frac{x-1}{a} = \frac{y+1}{b} = \frac{z-1}{c} = \lambda \ .... (i)$$ ½ m

According to the question $3a + 4b + 2c = 0$

$a - b + 2c = 0$ 1 m

Solving, $\dfrac{a}{10} = \dfrac{b}{-4} = \dfrac{c}{-7} = \mu$

$\Rightarrow \quad a = 10\mu\,,\; b = -4\mu\,, c = -7\mu$ ½ m.

(i) $\Rightarrow$ Equation of the line is

$$\frac{x-1}{10} = \frac{y+1}{-4} = \frac{z-1}{-7} \text{ [cartesian form]}$$ ½ m

Vector form, $\vec{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 7\hat{k}\right)$ ½ m

9.

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} H & R & HW \end{array} \\ \begin{bmatrix} 3 & 4 & 6 \\ 4 & 5 & 3 \\ 6 & 2 & 4 \end{bmatrix} \end{array} \begin{array}{c} \\ \begin{bmatrix} 2500 \\ 3100 \\ 5100 \end{bmatrix} \end{array}$$ 1 m

$$= \begin{bmatrix} 50500 \\ 40800 \\ 41600 \end{bmatrix}$$ 1 m

Hence money awarded by A = Rs. 50500

money awarded by B = Rs. 40800 1 m

money awarded by C = Rs. 41600

Respect for elders or Any relevant value 1 m

10. Given $\dfrac{x}{x-y} = \log a - \log(x-y)$ ½ m

Differentiating both sides and getting $[\because x \neq y]$

$$x - 2y + y\frac{dy}{dx} = 0$$ 2½ m

$$\Rightarrow \frac{dy}{dx} = \frac{2y-x}{y}$$

$$\Rightarrow \quad \frac{dy}{dx} = 2 - \frac{x}{y}$$ 1 m

11. $$I = \int \frac{dx}{x^3 \left(x^5 + 1\right)^{3/5}}$$

$$= \int \frac{dx}{x^3 \cdot x^3 \left(1 + \frac{1}{x^5}\right)^{3/5}}$$ 1½ m

Put $1 + \frac{1}{x^5} = t$

$$\Rightarrow \quad \frac{dx}{x^6} = -\frac{dt}{5}$$ 1 m

$$\therefore \quad I = -\frac{1}{5} \int t^{-3/5} dt = -\frac{1}{2} t^{2/5} + C$$ 1 m

$$= -\frac{1}{2} \left(1 + \frac{1}{x^5}\right)^{2/5} + C$$ ½ m

12. $$I = \int_2^4 |x-2|\,dx + \int_2^4 |x-3|\,dx + \int_2^4 |x-4|\,dx$$ ½ m

$$= \int_2^4 (x-2)\,dx + \int_2^3 -(x-3)\,dx + \int_3^4 (x-3)\,dx + \int_2^4 -(x-4)\,dx$$ 2 m

$$= \left[\frac{x^2}{2} - 2x\right]_2^4 - \left[\frac{x^2}{2} - 3x\right]_2^3 + \left[\frac{x^2}{2} - 3x\right]_3^4 - \left[\frac{x^2}{2} - 4x\right]_2^4$$ 1 m

$$= 5$$ ½ m

OR

$$I = \int_0^{\pi/4} \frac{\sec x}{1 + 2\sin^2 x}\,dx = \int_0^{\pi/4} \frac{\cos x}{\cos^2 x \left(1 + 2\sin^2 x\right)}\,dx$$

$$= \int_0^{\pi/4} \frac{\cos x}{(1-\sin x)(1+\sin x)(1+2\sin^2 x)}\, dx$$ 1 m

Put $\sin x = t \quad \Rightarrow \quad \cos x\, dx = dt$, when $x = 0, t = 0$

$$x = \frac{\pi}{4},\ t = \frac{1}{\sqrt{2}}$$ ½ m

$$\therefore \quad I = \int_0^{1/\sqrt{2}} \frac{dt}{(1-t)\ (1+t)\ (1+2t^2)}$$

$$\therefore \quad I = \int_0^{1/\sqrt{2}} \frac{dt}{6(1-t)} + \int_0^{1/\sqrt{2}} \frac{dt}{6(1+t)} + \int_0^{1/\sqrt{2}} \frac{2\ dt}{3(1+2t^2)}$$ 1 m

$$= \left[\frac{1}{6} \log \left|\frac{1+t}{1-t}\right| + \frac{\sqrt{2}}{3} \tan^{-1}\left(\sqrt{2}\ t\right)\right]_0^{\frac{1}{\sqrt{2}}}$$ 1 m

$$= \frac{1}{6} \log \left|\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}\right| + \frac{\sqrt{2}}{3} \tan^{-1}(1)$$

$$= \frac{1}{3} \log \left|\sqrt{2}+1\right| + \frac{\pi}{6\sqrt{2}} \quad or \quad \frac{1}{6} \log \left(3+2\sqrt{2}\right) + \frac{\pi}{6\sqrt{2}}$$ ½ m

13. $$I = \int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1-\sin 2x}{1-\cos 2x}\right) dx$$

$$= \int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1-2\sin\cos x}{2\sin^2 x}\right) dx$$

$$= \int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1}{2}\ \mathrm{cosec}^2 x - \cot x\right) dx$$ 1½ m

Put $\quad 2x = t \quad \Rightarrow \quad dx = \frac{dt}{2}$

when $x = \frac{\pi}{4},\ t = \frac{\pi}{2};\ x = \frac{\pi}{2},\ t = \pi$ 1 m

$$\therefore \quad I = \frac{1}{2} \int_{\pi/2}^{\pi} e^t \left(\frac{1}{2} \operatorname{cosec}^2 \frac{t}{2} - \cot \frac{t}{2} \right) dt$$

$$= -\frac{1}{2} \left[\cot \frac{t}{2} \cdot e^t \right]_{\frac{\pi}{2}}^{\pi}$$ 1 m

$$= \frac{e^{\pi/2}}{2}$$ ½ m

14. Let $\overrightarrow{OA} = 4\hat{i} + 8\hat{j} + 12\hat{k},\ \overrightarrow{OB} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

$\overrightarrow{OC} = 3\hat{i} + 5\hat{j} + 4\hat{k},\ \overrightarrow{OD} = 5\hat{i} + 8\hat{j} + 5\hat{k}$

$\overrightarrow{AB} = -2\hat{i} - 4\hat{j} - 6\hat{k},\ \overrightarrow{AC} = -\hat{i} - 3\hat{j} - 8\hat{k},\ \overrightarrow{AD} = \hat{i} - 7\hat{k}$ 1½ m

Now, $\left[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD} \right] = \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & -7 \end{vmatrix} = 0$ 2 m

$\therefore$ A, B, C, D are coplanar ½ m

15. LHS $= \sin \left[\cot^{-1} \left(\frac{2x}{1 - x^2} \right) + 2 \tan^{-1} x \right]$ 1 m

$$= \sin \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{2x}{1 - x^2} \right) + 2 \tan^{-1} x \right]$$ 1 m

$$= \sin \left[\frac{\pi}{2} - 2 \tan^{-1} x + 2 \tan^{-1} x \right]$$ 1 m

$= \sin \frac{\pi}{2} = 1 = \text{R.H.S}$ 1 m

OR

$$\tan^{-1}\left(\frac{\frac{x-5}{x-6}+\frac{x+5}{x+6}}{1-\frac{x-5}{x-6}\cdot\frac{x+5}{x+6}}\right) = \frac{\pi}{4}$$ 2 m

$$\Rightarrow \quad \frac{(x-5)(x+6)+(x+5)(x-6)}{x^2-36-x^2+25} = \tan\frac{\pi}{4}$$ 1 m

$$\Rightarrow \quad 2x^2 = 49$$ ½ m

$$\Rightarrow \quad x = \pm \frac{7}{\sqrt{2}}$$ ½ m

16. L.H.S. $= \begin{vmatrix} 1+a^2-b^2 & 2\,ab & -2b \\ 2\,ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

$R_1 \to R_1 + b\cdot R_3, \quad R_2 \to R_2 - a\,R_3$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -b\left(1+a^2+b^2\right) \\ 0 & 1+a^2+b^2 & a\left(1+a^2+b^2\right) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ 1+1 m

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ 1 m

Expanding and getting

$\Delta = \left(1+a^2+b^2\right)^3 = \text{R.H.S.}$ 1 m

17. $A^2 = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}$ 1½ m

$$A^2 - 5A + 4I = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - \begin{pmatrix} 10 & -5 & 5 \\ -5 & 10 & -5 \\ 5 & -5 & 10 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$= O$ 1 m

Pre multiplying by A^{-1} and getting $A^{-1} = \frac{1}{4}(5I - A)$ ½ m

and $A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$ 1 m

OR

$A = IA$ 1 m

$$\Rightarrow \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$R_3 \rightarrow R_3 - 3R_1$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -9 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix} A$$

$R_1 \to R_1 - 2R_2, \quad R_3 \to R_3 + 5R_2$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{pmatrix} A$$

$R_1 \to R_1 + R_3, \quad R_2 \to R_2 - 2R_3$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{pmatrix} A$$ [operating Row wise to reach at this step] 2½ m

$$A^{-1} = \begin{pmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{pmatrix}$$ ½ m

18. A Candidate who has made an attempt to solve the question to be given 4 marks 4 m

19. $y = -x^3 \log x$ ½ m

$\frac{dy}{dx} = -x^2(1 + 3\log x)$ 1 m

$\frac{d^2y}{dx^2} = -(5x + 6x\log x)$ 1 m

L.H.S. $= x[-(5x + 6x\log x)] + 2x^2(1 + 3\log x) + 3x^2$ 1 m

$= 0$ ½ m

$=$ R.H.S.

OR

$f(x) = (x-4)](x-6)(x-8)$

$= x^3 - 18x^2 + 104x - 192$

Being a polynomial function $f(x)$ is continuous in [4, 10] and differentiable in (4, 10) with

$f'(x) = 3x^2 - 36x + 104$ 1+1 m

$\exists\ c \in (4,10) \cdot$ such that $\quad f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$\Rightarrow \quad 3c^2 - 36c + 104 = 8$ 1½ m

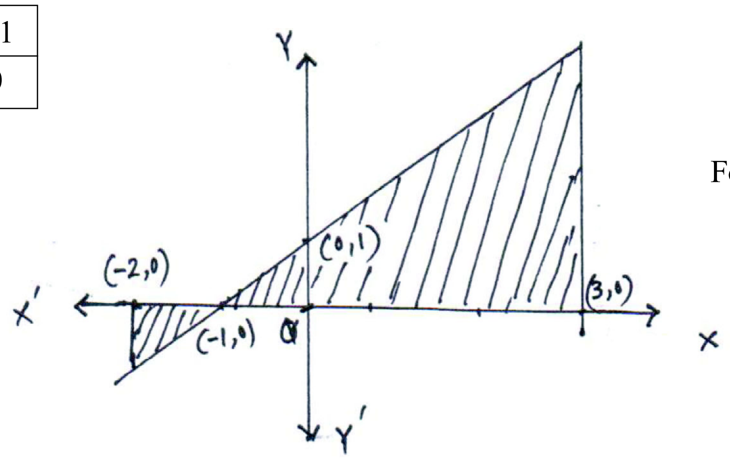
$\Rightarrow \quad c = 4, 8 \qquad ;\ c = 4 \notin (4, 10)$

$\therefore \quad c = 8$: verifies the theoren ½ m

SECTION - C

20. $y = x + 1, \quad x = -2, \quad x = 3$

x	0	−1
y	1	0



For correct figure 1 m

Reqd area $= \left| \int_{-2}^{-1} (x+1)\,dx \right| + \int_{-1}^{3} (x+1)\,dx$ 2 m

$= \left| \left(\dfrac{x^2}{2} + x \right)_{-2}^{-1} \right| + \left(\dfrac{x^2}{2} + x \right)_{-1}^{3}$ 2 m

$= \dfrac{17}{2}$ sq. units 1 m

21. $(y - \sin x)\,dx + (\tan x)\,dy = 0 \Rightarrow \dfrac{dy}{dx} + \cot x\, y = \cos x$ 1 m

Linear diff. equ. with $P = \cot x, \ Q = \cos x$

I.F. $= \sin x$ 1 m

Solution is $y \cdot \sin x = \int \cos x \cdot \sin x \, dx + c$

$$= -\frac{1}{4} \cos 2x + c$$ 2 m

when $x = 0, \; y = 0 \;\Rightarrow\; c = \frac{1}{4}$ 1 m

Particular solution is

$$y \sin x = \frac{1}{4}\left(-\cos 2x + 1\right) = \frac{\sin^2 x}{2}$$

$$\Rightarrow \quad y = \frac{1}{2} \sin x$$ 1 m

22. Let x denote no. of heads

here $p = \frac{1}{2}, \quad q = \frac{1}{2}$ 1 m

$$P(x = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r}$$

$$= {}^nC_r \left(\frac{1}{2}\right)^n$$ 1 m

Now $P(x = 1) = {}^nC_1 \left(\frac{1}{2}\right)^n$

$P(x = 2) = {}^nC_2 \left(\frac{1}{2}\right)^n$ 1½ m

$P(x = 3) = {}^nC_3 \left(\frac{1}{2}\right)^n$

According to the question

$$2. \; {}^nC_2 \left(\frac{1}{2}\right)^n = \left({}^nC_1 + {}^nC_3\right)\left(\frac{1}{2}\right)^n$$ 2 m

$\Rightarrow \quad n = 2 \text{ or } 7$ ½ m

n can not be 2 Hence $n = 7$

23. $d \cdot r's$ of first line : $k-5, 1, 2k+1$ 1 m

$d \cdot r's$ of 2nd line : $-1, k, 5$ 1 m

$\because$ lines are $\perp$ $\therefore$ $-1(k-5)+k(1)+5(2k+1)=0$

$\Rightarrow \quad k=-1$ 1 m

Eqns of lines become $\frac{x+3}{-6}=\frac{y-1}{-1}=\frac{z-5}{-1}$ and $\frac{x+2}{-1}=\frac{y-2}{-1}=\frac{z}{5}$ 1 m

Eqn of plane containing these two lines is

$$\begin{vmatrix} x+2 & y-2 & z \\ -6 & 1 & -1 \\ -1 & -1 & 5 \end{vmatrix} = 0$$ 1 m

$\Rightarrow \quad 4x+31y+7z=54$ 1 m

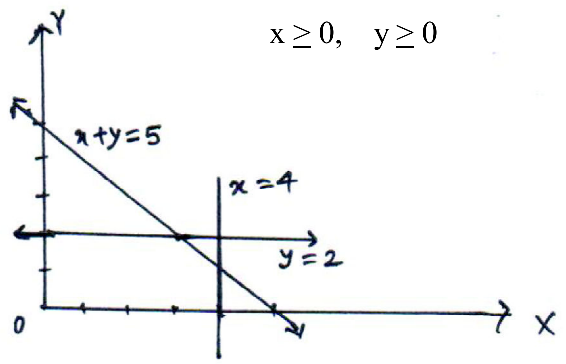
24. Let x kg of B_1 and y kg of B_2 is taken

then to minimize $Z = 5x+8y$ 1 m

subject to the following constraints 3 m

$x+y=5, \quad x \leq 4, \quad y \geq 2$

$x \geq 0, \quad y \geq 0$



Graph 2 m

25. $(a, b) * (c, d) = (a + c, b + d) \quad \forall\ a, b, c, d \in R$

Since $a + c \in R$ and $b + d \in R \Rightarrow (a + c, b + d) \in R \times R$ 1½ m

i.e. '*' is binary operation

For commutative

consider $(c, d) * (a, b) = (c + a,\ d + b)$

$= (a + c,\ b + d)$

$= (a, b) * (c, d)$ 1½ m

$\Rightarrow$ '*' is commutative

For Associative

Let $(a, b),\ (c, d),\ (e, f) \in R \times R = A$

$[(a, b) * (c, d)] * (e, f) = (a + c,\ b + d) * (e, f)$

$= (a + c + e, b + d + f)$ ....(i)

again $(a, b) * [(c, d)] * (e, f)] = (a, b) * (c + e, d + f)$

$= (a + c + e, b + d + f)$ ....(ii) 1½ m

(i) & (ii) $\Rightarrow$ '*' is associative

For identity element

Let $(e_1, e_2) \in R \times R$ be the identity element (if exists)

then $(a, b) * (e_1, e_2) = (a, b) = (e_1, e_2) * (a, b)$

$\Rightarrow (e_1, e_2) = (0, 0) \in R \times R$ 1½ m

OR

$f(x) = x^2 - x ; \quad x \in \{-1, 0, 1, 2\}$

$f(-1) = 2,\ f(0) = 0,\ f(1) = 0,\ f(2) = 2$

$\therefore\ f = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

$g(x) = 2\left|x - \frac{1}{2}\right| - 1 \quad \forall\ x \in \{-1, 0, 1, 2\}$

$g(-1) = 2,\ g(0) = 0,\ g(1) = 0,\ g(2) = 2$

$\therefore\ g = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

$(g\,of)(x) = g(f(-1),\ g(f(0),\ g(f(1),\ g(f(2)\ \ \forall\ x \in A$

$= 2, 0, 0, 2$

$\therefore\ \ g\,of = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

Hence $f = g = g\,o\,f$

26. Given curve cuts the x – axis when $y = 0$ ½ m

when $y = 0,\ x = 7,$ hence point is $(7, 0)$ ½ m

$$\frac{dy}{dx} = \frac{1 - y(2x-5)}{x^2 - 5x + 6}$$ 2½ m

$$\left.\frac{dy}{dx}\right]_{(7,0)} = \frac{1}{20}$$ ½ m

Equation of the tangent is $y - 0 = \frac{1}{20}(x - 7)$ 1 m

$\Rightarrow\ \ x - 20y = 7$

Equation of the normal is $y - 0 = -20(x - 7)$ 1 m

$\Rightarrow\ \ 20x + y = -7$

OR

$f(x) = \cos^2 x + \sin x,\ \ x \in [0, \pi]$

$f'(x) = \cos x\,(-2\sin x + 1)$ 1 m

For extremum, $f'(x) = 0 \Rightarrow x = \frac{\pi}{2}$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}$ 1½ m

Now $f(0) = 1,\ f\left(\frac{\pi}{6}\right) = \frac{5}{4},\ f\left(\frac{\pi}{2}\right) = 1,\ f\left(\frac{5\pi}{6}\right) = \frac{5}{4},\ f(\pi) = 1$ 1½ m

Absolute max. is $\frac{5}{4}$ at $x = \frac{\pi}{2}$ and $\frac{5\pi}{6}$ 1 m

Absolute min. is 1 at $x = 0,\ \frac{\pi}{6}$ and π 1 m