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Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65/1/F, 65/2/F, 65/3/F

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/F

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\vec{a} + \vec{b} = 6\hat{i} + \hat{k}$ ½ m

$\therefore$ Reqd. unit vector $= \frac{6}{\sqrt{37}}\hat{i} + \frac{1}{\sqrt{37}}\hat{k}$ ½ m

2. Reqd. area $= \left|\vec{a} \times \vec{b}\right|$ ½ m

$\therefore \left|12\hat{i} - 4\hat{j} + 8\hat{k}\right| = \sqrt{144 + 16 + 64} = \sqrt{224}$ or $4\sqrt{14}$ sq. units ½ m

3. Getting x – intercept $= \frac{5}{2}$, y – intercept $= 5$, z – intercept $= -5$ ½ m

$\therefore$ Their sum $= \frac{5}{2}$ ½ m

4. co – factor of $a_{21} = 3$ 1 m

5. Degree = Order = 2 any one correct ½ m

$\therefore$ Degree + order = 4 ½ m

6. $2^y\, dy = dx \Rightarrow \frac{2^y}{\log 2} = x + c$ ½+½ m

SECTION - B

7. Getting $A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$ 1 m

$4A - 3I = \begin{bmatrix} 8-3 & -4 \\ -4 & 8-3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} = A^2$ 1 m

$A^2 = 4A - 3I$ ....(i)

Multiply both sides by A^{-1} ½ m

$A = 4I - 3A^{-1}$ or $A^{-1} = \frac{1}{3}(4I - A) = \frac{1}{3}\begin{pmatrix} 4-2 & 0+1 \\ 0+1 & 4-2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ 1½ m

OR

$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a^2+b & a-1 \\ b(a-1) & b+1 \end{bmatrix}$ 1½ m

$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a) - 2(2+b) & 4 \end{pmatrix}$ .... (i) 1½ m

$A^2 + B^2 = \begin{pmatrix} a^2+b-1 & a-1 \\ b(a-1) & b \end{pmatrix}$ ....(ii)

Equating (i) and (ii), we get $b = 4, a = 1$ 1 m

8. Using $C_1 \to C_1 + C_2 + C_3$ and taking $a^2 + a + 1$ common from C_1

$\Delta = (a^2 + a + 1)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}$, using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ 1½ m

$$\Delta = \left(a^2+a+1\right)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & a(a-1) & (1-a)(1+a) \end{vmatrix} = \left(a^2+a+1\right)(1-a)^2\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & 1+a \end{vmatrix}$$ 1½ m

$$\left.\begin{aligned} &= \left(a^2+a+1\right)(1-a)^2\left(1+a+a^2\right) \\ &= \left[(1-a)\left(1+a+a^2\right)\right]^2 = \left(1-a^3\right)^2 \end{aligned}\right\}$$ 1 m

9. Let $x+a=t \Rightarrow dx = dt$ and $x = t-a \Rightarrow x-a = t-2a$ 1 m

$$\therefore\ I = \int \frac{\sin(t-2a)\,dt}{\sin t} = \int \frac{(\sin t \cos 2a - \cos t \cdot \sin 2a)\,dt}{\sin t}$$ 1 m

$$= \cos 2a \int dt - \sin 2a \int \cot t\, dt = \cos 2a\, t - \sin 2a . \log|\sin t| + c$$ 1 m

$$= \cos 2a\,(x+a) - \sin 2a \log|\sin(x+a)| + c$$ 1

OR

consider $\dfrac{x^2}{\left(x^2+4\right)\left(x^2+9\right)}$ · Let $x^2 = t$ ½ m

$$\therefore\ \frac{t}{(t+4)(t+9)} = -\frac{4}{5}\ \frac{1}{t+4} + \frac{9}{5}\ \frac{1}{t+9}$$ 1 m

$$I = \int \frac{t\ \ dt}{(t+4)(t+9)} = -\frac{4}{5}\int\frac{dt}{t+4} + \frac{9}{5}\int\frac{dt}{t+9}$$ ½ m

$$= \frac{-4}{5}\log|t+4| + \frac{9}{5}\log|t+9| + c$$ 1½ m

$$\therefore\ I = -\frac{4}{5}\log\left|x^2+4\right| + \frac{9}{5}\log\left|x^2+9\right| + c$$ ½ m

10. Writing given integral as 1 m

$$I = \int_{-\pi/2}^{0}\frac{\cos x}{1+e^x}dx + \int_{0}^{\pi/2}\frac{\cos x}{1+e^x}dx$$ Let $x = -t$, $dx = -dt$ 1½ m

when $x = -\dfrac{\pi}{2}$, $t = \pi/2$

$x = 0$, $t = 0$

$$\therefore\ I = \int_0^{\frac{\pi}{2}} \frac{e^t \cos t}{1+e^t}\, dt + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\, dx = \int_0^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x}\, dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\, dx$$ 1½ m

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(1+e^x\right)\cos x\, dx}{\left(1+e^x\right)} = \int_0^{\frac{\pi}{2}} \cos x\, dx = \left(\sin\ x\right)_0^{\frac{\pi}{2}} = 1$$ 1 m

11. Let B_1, B_2, B_3 be the events that the bolts produced by machines E_1, E_2, E_3 and A be the event that the selected bulb is defective ½ m

$$\left.\begin{aligned} &\therefore\ P\left(B_1\right) = \frac{1}{2},\ P\left(B_2\right) = P\left(B_3\right) = \frac{1}{4} \\ &P\left(A/B_1\right) = \frac{1}{25},\ P\left(A/B_2\right) = \frac{1}{25},\ P\left(A/B_3\right) = \frac{1}{20} \end{aligned}\right\}$$ 1½ m

$$P\left(A\right) = \sum_{c=1}^{3} P\left(B_c\right) P\left(A/B_c\right) = \frac{1}{2}\times\frac{1}{25}+\frac{1}{4}\times\frac{1}{20}+\frac{1}{4}\times\frac{1}{25}$$ 1+½ m

$$= \frac{17}{400}$$ ½ m

OR

$$\therefore\ P\left(x=3\right) = \frac{1}{6}\times\frac{1}{5}\times 2 = \frac{1}{15},\ P\left(x=4\right) = \frac{2}{6}\times\frac{1}{5}\times 2 = \frac{2}{15}$$

Simlarly $P\left(x=5\right) = \frac{3}{15},\ P\left(x=6\right) = \frac{4}{15},\ P\left(x=7\right) = \frac{5}{15}$

Prob. distribution is

x :	3	4	5	6	7
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

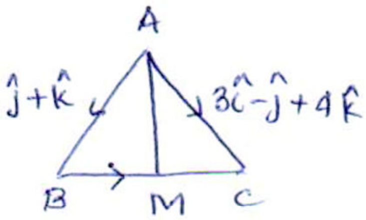
2 m

$x \cdot P(x)$:	$\frac{3}{15}$	$\frac{8}{15}$	$\frac{15}{15}$	$\frac{24}{15}$	$\frac{35}{15}$
$x^2 P(x)$:	$\frac{9}{15}$	$\frac{32}{15}$	$\frac{75}{15}$	$\frac{144}{15}$	$\frac{245}{15}$

$$\text{Mean} = \sum x_i \cdot P(x_i) = \frac{85}{15} = \frac{17}{3}$$ 1 m

$$\text{Variance} = \sum x_i^2\, P(x_i) - (\text{Mean})^2 = \frac{101}{3} - \frac{289}{9} = \frac{14}{9}$$ 1 m

12.



$$\vec{BC} = \left(3\hat{i} - \hat{j} + 4\hat{k}\right) - \left(\hat{j} + \hat{k}\right) = 3\hat{i} - 2\hat{j} + 3\hat{k}$$ 1½ m

$$\therefore\ \vec{BM} = \frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}$$ 1 m

$$AM = \left|\hat{j} + \hat{k} + \frac{3\hat{i} - 2\hat{j} + 3\hat{k}}{2}\right| = \left|\frac{3\hat{i} + 5\hat{k}}{2}\right| = \frac{\sqrt{34}}{2}$$ 1½ m

13. Any plane through given point is $a(x-3) + b(y-6) + c(z-4) = 0$..........(i) 1 m

with $a + 5b + 4c = 0$ ....(A) ½ m

(i) passes through $(3, 2, 0)$ $\Rightarrow -4b - 4c = 0$ or $b + c = 0$ ....(B) ½ m

From (A) and (B) $a + b + (4b + 4c) = 0 \Rightarrow a = -b$ 1 m

$\therefore\ a = -b = c$

$\therefore$ Required eqn. of plane is $x - y + z - 1 = 0$ 1 m

14. $\text{LHS} = \tan^{-1}\left(\frac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}\frac{2\cos\theta}{\sin^2\theta}$ 2 m

$\therefore\ \tan^{-1}\frac{2\cos\theta}{\sin^2\theta} = \tan^{-1}\left(\frac{2}{\sin\theta}\right)$ 1 m

$\Rightarrow\ \cot\theta = 1 \text{ or } \theta = \pi/4$ 1 m

OR

The given equation can be written

$\left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)+\left(\tan^{-1}4-\tan^{-1}3\right)+\ldots..+\tan^{-1}(n+1)-\tan^{-1}n=\tan^{-1}\theta$ 2 m

$\Rightarrow\ \tan^{-1}(n+1)-\tan^{-1}1 = \tan^{-1}\theta$ 1 m

$\Rightarrow\ \tan^{-1}\frac{n+1-1}{1+(n+1)} = \tan^{-1}\theta \Rightarrow \theta = \frac{n}{n+2}$ 1 m

15. $9y^2 = x^3 \Rightarrow 18y\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \text{slope of tangent} = \frac{x^2}{6y}$

$\therefore$ Slope of normal $= -\frac{6y}{x^2}$ 1+½ m

As the intercepts by normal on both axes are equal

$\therefore$ Slope of normal $= \pm 1 \Rightarrow \frac{-6y}{x^2} = \pm 1 \Rightarrow y = \pm\frac{x^2}{6}$ 1 m

$\therefore\ 9\left(\frac{x^4}{36}\right) = x^3 \Rightarrow x = 4 \text{ and } y^2 = \frac{64}{9} \Rightarrow y = \pm 8/3$ 1 m

$\therefore$ The points are $\left(4, \frac{8}{3}\right), \left(4, -8/3\right)$ ½ m

16. $\frac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left[1+\frac{x}{\sqrt{1+x^2}}\right] = \frac{n}{\sqrt{1+x^2}}\left[x+\sqrt{1+x^2}\right]^n = \frac{ny}{\sqrt{1+x^2}}$ 1½ m

$\therefore\ \sqrt{1+x^2}\,\frac{dy}{dx} = ny\ldots\ldots\ldots\ldots(i)$ ½ m

$\therefore\ \sqrt{1+x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{x}{\sqrt{1+x^2}} = n\frac{dy}{dx}$ 1 m

$\Rightarrow\ \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{1+x^2}\,\frac{dy}{dx} = n\cdot ny$ (from (i))

$= n^2y$ 1 m

17. L H D at $x=1$: $\lim\limits_{x\to 1^-}\left(\frac{x-1}{x-1}\right) = 1$

R H D at $x=1$, $\lim\limits_{x\to 1^+}\frac{2-x-1}{x-1} = -1$ 2 m

$\therefore$ f is not differentiable at $x=1$

L H D at $x=2$, $\lim\limits_{x\to 2^-}\frac{2-x-0}{x-2} = -1$

R H D at $x=2$, $\lim\limits_{x\to 2^+}\frac{-2+3x-x^2}{(x-2)} = \lim\limits_{x\to 2^+} -\frac{(x-1)(x-2)}{(x-2)} = -1$ 2 m

$\therefore$ f is diff. at $x=2$

18. Communication Matrix $A = \begin{array}{c}\begin{pmatrix}140\\200\\150\end{pmatrix}\end{array}\begin{array}{l}\text{Telephone}\\\text{House calls}\\\text{Letters}\end{array}$

$$\begin{array}{lccc} & \text{Tele} & \text{House calls} & \text{Letters} \\ \text{Cost Matrix } B = & \multicolumn{3}{c}{\begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix}} \end{array} \begin{array}{l} \text{City x} \\ \text{City y} \end{array}$$

$$\therefore \text{ Total cost Matrix } = \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} = \begin{pmatrix} 990000 \\ 2120000 \end{pmatrix}$$ 3 m

any relevant value 1 m

19. $I = \int e^{2x} \sin(3x+1)\,dx = \left[\frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \int e^{2x} \cos(3x+1)\,dx\right]$ 1½ m

$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \cdot \frac{e^{2x}}{2} \cdot \cos(3x+1) - \frac{9}{4} \int e^{2x} \sin(3x+1)\,dx \Big]$ 1 m

$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) - \frac{9}{4} I$ 1 m

$$\left.\begin{array}{l} \frac{13}{4} I = \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) \\ I = \frac{4}{13}\left[\frac{e^{2x}}{2}\left(-\frac{3}{2}\cos(3x+1) + \sin(3x+1)\right)\right] + c \end{array}\right\}$$ ½ m

SECTION - C

20. Let $x_1, x_2, \in R$ such that $f(x_1) = f(x_2) \Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ 1½+½ m

$\Rightarrow 4(x_1 - x_2)[x_1 + x_2 + 3] = 0 \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one – one

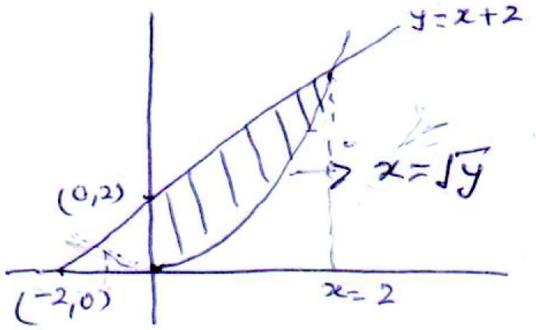
f is clearly onto and hence intertible 1 m

Let y be an arbitrary element of S

$f(x) = y = 4x^2 + 12x + 15 = (2x+3)^2 + 6$ 1 m

$\therefore f^{-1} : R \to S$ is given by $f^{-1}(y) = \left(\frac{\sqrt{y-6} - 3}{2}\right)$ 2 m

21.



Correct Figure 1m

Points of intersection

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$ (-1 is rejected) 1½ m

$\therefore$ Reqd. area $= \int_0^2 \left\{(x+2) - x^2\right\} dx$ 1½ m

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_0^2$

$= \left(2 + 4 - \frac{8}{3}\right) = \frac{10}{3}$ sq.units 2 m

22. Let $z = ax + by$, also $xy = c^2 \Rightarrow y = \frac{c^2}{x}$

$\therefore \; z = ax + \frac{bc^2}{x}$ ½ + ½ m

$\therefore \; \frac{dz}{dx} = a + bc^2\left(\frac{-1}{x^2}\right), \; \frac{dz}{dx} = 0 \Rightarrow bc^2 = ax^2$

or $x = \sqrt{\frac{b}{a}}\, c$ 1½ m

showing $\frac{d^2z}{ax^2}$ at $x = \sqrt{\frac{b}{a}}\, c > 0 \Rightarrow$ minima 1½ m

$y = \frac{c^2}{x} = \frac{c^2}{c}\sqrt{\frac{a}{b}} = c\sqrt{\frac{a}{b}}$ 1 m

$\therefore$ minimum $z = a\sqrt{\frac{b}{a}}\, c + bc\sqrt{\frac{a}{b}}\, c = 2\sqrt{ab}\, c$ 1 m

OR

$y = x^2 + 7x + 2\ ,\ 3x - y - 3 = 0$ ....(i)

$\therefore\ 3x - \left(x^2 + 7x + 2\right) - 3 = 0$ 1 m

Distance of (x, y) from (i)

$$D = \left| \frac{3x - \left(x^2 + 7x + 2\right) - 3}{\sqrt{10}} \right| \text{ or } D = \left| \frac{\left(-x^2 - 4x - 5\right)}{\sqrt{10}} \right| = \left| \frac{(x+2)^2 + 1}{\sqrt{10}} \right|$$ 2 m

$$\frac{dD}{dx} = \frac{2}{\sqrt{10}}\ (x+2),\ \frac{dD}{dx} = 0 \text{ at } x = -2$$ 1 m

$$\frac{d^2D}{dx^2} > 0 \Rightarrow \text{ minima}$$ 1 m

$\therefore$ D is minimum at $x = -2$

at $x = -2$, $y = -8$

$\therefore$ The required pt. on the parabola is $(-2, -8)$ 1 m

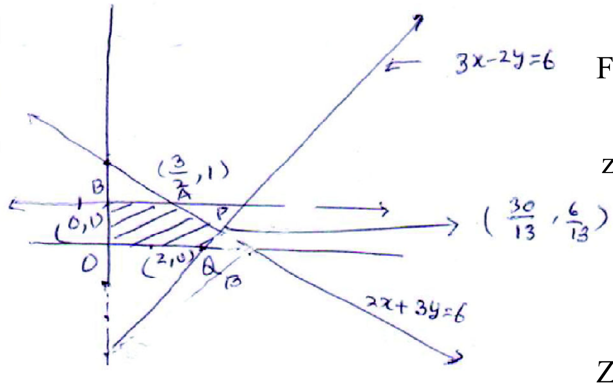
23. 

Figure 3 m

Feasible region is B A PQO 1 m

$$z_B = 9,\ z_P = \frac{240}{13} + \frac{54}{13} = \frac{294}{13}$$

$$= 22\frac{8}{13}$$ 1 m

$Z_Q = 16$

$$\therefore\ Z \text{ is maximum at } \left(\frac{30}{13}, \frac{6}{13}\right)$$

$$\text{and maximum value} = 22\frac{8}{13}$$ 1 m

24. Any line through $(1,-2,3)$ with d. r's as 2, 3 – 6 is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$ 1 ½ m

$$\therefore\ x = 2\lambda+1,\ y = 3\lambda-2,\ z = -6\lambda+3$$ 1½ m

It lies on the plane $x - y + z = 5$

$$\therefore\ 2\lambda+1-3\lambda+2-6\lambda+3 = 5$$

$$\Rightarrow\ \lambda = \frac{1}{7}$$ 1 m

Reqd. point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$ 1 m

$$\therefore\ \text{Reqd distance} = \sqrt{\left(\frac{9}{7}-1\right)^2+\left(-\frac{11}{7}+2\right)^2+\left(\frac{15}{7}-3\right)^2} = \frac{1}{7}\sqrt{4+9+36} = \frac{7}{7} = 1$$ 1 m

25. $$\frac{dy}{dx} = \frac{2x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)}{y - x\cos\left(\frac{y}{x}\right)} = \frac{2\sin\left(\frac{y}{x}\right) - \frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{y}{x} - \cos\left(\frac{y}{x}\right)}$$ 1 m

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$$\therefore\ v + x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v} \Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v^2}{v - \cos v}$$ 1 m

$$\Rightarrow \frac{v-\cos v}{-2\sin v + v^2} = \frac{-dx}{x} \text{ or } \frac{1}{2}\left[\frac{2v - 2\cos v}{-2\sin v + v^2}\right]dv = -\frac{dx}{x}$$ 1 m

$$\left.\begin{aligned}&\Rightarrow \ \frac{1}{2}\ \log\left|v^2-2\sin v\right| = -\log x+\log c\\ &\text{or } \log\left|\sqrt{v^2-2\sin v}\right| = \log c-\log x\end{aligned}\right\}\qquad 1\text{ m}$$

$$\sqrt{v^2-2\sin v} = \frac{c}{x}$$

$$\text{or } x\sqrt{\frac{y^2}{x^2} - 2\sin\frac{y}{x}} = c \qquad \tfrac{1}{2}\text{ m}$$

$$y^2-2x^2\sin\left(\frac{y}{x}\right) = c' \qquad \tfrac{1}{2}\text{ m}$$

OR

$$\left(\sqrt{(1+x^2)(1+y^2)}\right)dx+xy\,dy=0$$

$$\Rightarrow \ \frac{y}{\sqrt{1+y^2}}\ dy+\frac{\sqrt{1+x^2}}{x}\ dx = 0 \qquad 1\text{ m}$$

$$\frac{1}{2}\int\frac{2y}{\sqrt{1+y^2}}\,dy+\int\frac{\sqrt{1+x^2}}{x}\,dx=0$$

$$\sqrt{1+y^2} + \int\frac{(1+x^2)}{x\sqrt{1+x^2}}\,dx = c \qquad 1\tfrac{1}{2}\text{ m}$$

$$I_1 = \int\frac{1}{x\sqrt{1+x^2}}\ dx + \int\frac{x}{\sqrt{1+x^2}}\ dx = I_2+\sqrt{1+x^2} \qquad 1\text{ m}$$

For I_2, Let $x=\frac{1}{t}$, $dx = \frac{-1}{t^2}\,dt$ 1 m

$$I_2 = \int\frac{-1}{t^2\cdot\frac{1}{t}\sqrt{1+\frac{1}{t^2}}}\ dt = -\int\frac{dt}{\sqrt{t^2+1}} = -\log\left[t+\sqrt{1+t^2}\right] \qquad 1\text{ m}$$

$$= -\log\left[\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right] = -\log\left[\frac{1+\sqrt{1+x^2}}{x}\right]$$

$$\therefore \text{ The solution is } \sqrt{1+x^2}+\sqrt{1+y^2} - \log\left(\frac{1+\sqrt{1+x^2}}{2}\right) = c$$ ½ m

26. $P(\text{Doublet}) = \frac{1}{6}$, $P(\text{not a doublet}) = \frac{5}{6}$

The random variate x can take values 0, 1, 2, 3, 4 1 m

x	0	1	2	3	4	
P(x)	$\left(\frac{5}{6}\right)^4$	$4\ \frac{1}{6}\left(\frac{5}{6}\right)^3$	$6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$	$4\left(\frac{1}{6}\right)^3\frac{5}{6}$	$\left(\frac{1}{6}\right)^4$	
	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$	2½ m

$$\text{Mean} = \sum x\,P(x) = \frac{500+300+60+4}{1296} = \frac{864}{1296} = \frac{2}{3}$$ 1 m

$$\sum x^2\,P(x) = \frac{500+600+180+16}{1296} = \frac{1296}{1296} = 1$$ 1 m

$$\therefore \text{ Variance } = 1-\left(\frac{2}{3}\right)^2 = \frac{5}{9}$$ ½ m

QUESTION PAPER CODE 65/2/F

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SECTION - A

Marks

1. co – factor of $a_{21} = 3$ 1 m

2. Degree = Order = 2 any one correct ½ m

$\therefore$ Degree + order = 4 ½ m

3. $2^y \, dy = dx \Rightarrow \dfrac{2^y}{\log 2} = x + c$ ½+½ m

4. $\vec{a} + \vec{b} = 6\hat{i} + \hat{k}$ ½ m

$\therefore$ Reqd. unit vector $= \dfrac{6}{\sqrt{37}}\hat{i} + \dfrac{1}{\sqrt{37}}\hat{k}$ ½ m

5. Reqd. area $= \left| \vec{a} \times \vec{b} \right|$ ½ m

$\therefore \left| 12\hat{i} - 4\hat{j} + 8\hat{k} \right| = \sqrt{144 + 16 + 64} = \sqrt{224}$ or $4\sqrt{14}$ sq. units ½ m

6. Getting x – intercept $= \dfrac{5}{2}$, y – intercept = 5, z – intercept = – 5 ½ m

$\therefore$ Their sum $= \dfrac{5}{2}$ ½ m

SECTION - B

7. Writing given integral as 1 m

$$I = \int_{-\pi/2}^{0} \frac{\cos x}{1+e^x}\,dx + \int_{0}^{\pi/2} \frac{\cos x}{1+e^x}\,dx$$

Let $x = -t$, $dx = -dt$ 1½ m

when $x = -\frac{\pi}{2}$, $t = \pi/2$

$x = 0$, $t = 0$

$$\therefore\ I = \int_{0}^{\frac{\pi}{2}} \frac{e^t \cos t}{1+e^t}\,dt + \int_{0}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx = \int_{0}^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x}\,dx + \int_{0}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx$$ 1½ m

$$I = \int_{0}^{\frac{\pi}{2}} \frac{\left(1+e^x\right)\cos x\,dx}{\left(1+e^x\right)} = \int_{0}^{\frac{\pi}{2}} \cos x\,dx = \left(\sin x\right)_0^{\frac{\pi}{2}} = 1$$ 1 m

8. Let B_1, B_2, B_3 be the events that the bolts produced by machines E_1, E_2, E_3 and A be the event that the selected bulb is defective ½ m

$$\therefore\ P(B_1) = \frac{1}{2},\ P(B_2) = P(B_3) = \frac{1}{4}$$

1½ m

$$P\left(A/B_1\right) = \frac{1}{25},\ P\left(A/B_2\right) = \frac{1}{25},\ P\left(A/B_3\right) = \frac{1}{20}$$

$$P(A) = \sum_{c=1}^{3} P(B_c)\,P\left(A/B_c\right) = \frac{1}{2}\times\frac{1}{25} + \frac{1}{4}\times\frac{1}{20} + \frac{1}{4}\times\frac{1}{25}$$ 1+½ m

$$= \frac{17}{400}$$ ½ m

OR

$\therefore\ P(x=3)=\frac{1}{6}\times\frac{1}{5}\times 2=\frac{1}{15},\ P(x=4)=\frac{2}{6}\times\frac{1}{5}\times 2=\frac{2}{15}$

Simlarly $P(x=5)=\frac{3}{15},\ P(x=6)=\frac{4}{15},\ P(x=7)=\frac{5}{15}$

Prob. distridution is

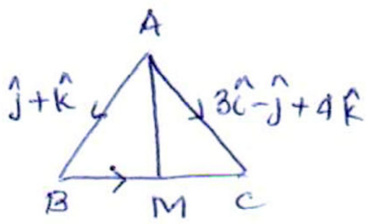
x :	3	4	5	6	7
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$
x . P (x) :	$\frac{3}{15}$	$\frac{8}{15}$	$\frac{15}{15}$	$\frac{24}{15}$	$\frac{35}{15}$
x^2 P (x) :	$\frac{9}{15}$	$\frac{32}{15}$	$\frac{75}{15}$	$\frac{144}{15}$	$\frac{245}{15}$

2 m

Mean $=\sum x_1\cdot P(x_1)=\frac{85}{15}=\frac{17}{3}$ 1 m

Variance $=\sum x_i^2\ P(x_i)-\ (\text{Mean})^2=\frac{101}{3}-\frac{289}{9}=\frac{14}{9}$ 1 m

9.



$\vec{BC}=\left(3\hat{i}-\hat{j}+4\hat{k}\right)-\left(\hat{j}+\hat{k}\right)=3\hat{i}-2\hat{j}+3\hat{k}$ 1½ m

$\therefore\ \vec{BM}=\frac{3}{2}\hat{i}-\hat{j}+\frac{3}{2}\hat{k}$ 1 m

$AM=\left|\hat{j}+\hat{k}+\frac{3\hat{i}-2\hat{j}+3\hat{k}}{2}\right|=\left|\frac{3\hat{i}+5\hat{k}}{2}\right|=\frac{\sqrt{34}}{2}$ 1½ m

10. Any plane through given point is $a(x-3)+b(y-6)+c(z-4)=0$..........(i) 1 m

with $a+5b+4c=0$ ....(A) ½ m

(i) passes through (3, 2, 0) $\Rightarrow\ -4b-4c=0$ or $b+c=0$ ....(B) ½ m

From (A) and (B) $a+b+(4b+4c) = 0 \Rightarrow a=-b$ 1 m

$\therefore\ a=-b=c$ 1 m

$\therefore$ Required eqn. of plane is $x-y+z-1=0$

11. $\text{LHS}=\tan^{-1}\left(\dfrac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta}$ 2 m

$\therefore\ \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta} = \tan^{-1}\left(\dfrac{2}{\sin\theta}\right)$ 1 m

$\Rightarrow\ \cot\theta=1$ or $\theta=\pi/4$ 1 m

OR

The given equation can be written

$(\tan^{-1}2-\tan^{-1}1)+(\tan^{-1}3-\tan^{-1}2)+(\tan^{-1}4-\tan^{-1}3)+.....+\tan^{-1}(n+1)-\tan^{-1}n=\tan^{-1}\theta$ 2 m

$\Rightarrow\ \tan^{-1}(n+1)-\tan^{-1}1 = \tan^{-1}\theta$ 1 m

$\Rightarrow\ \tan^{-1}\dfrac{n+1-1}{1+(n+1)} = \tan^{-1}\theta \Rightarrow \theta = \dfrac{n}{n+2}$ 1 m

12. Getting $A^2=\begin{bmatrix}5 & -4\\ -4 & 5\end{bmatrix}$ 1 m

$4A-3I = \begin{bmatrix}8-3 & -4\\ -4 & 8-3\end{bmatrix} = \begin{bmatrix}5 & -4\\ -4 & 5\end{bmatrix} = A^2$ 1 m

$A^2 = 4A - 3I$ ....(i)

Multiply both sides by A^{-1} ½ m

$$A = 4I - 3A^{-1} \text{ or } A^{-1} = \frac{1}{3}(4I - A) = \frac{1}{3}\begin{pmatrix} 4-2 & 0+1 \\ 0+1 & 4-2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$ 1½ m

OR

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \ B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a^2+b & a-1 \\ b(a-1) & b+1 \end{bmatrix}$$ 1½ m

$$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a)-2(2+b) & 4 \end{pmatrix} \text{ .... (i)}$$ 1½ m

$$A^2 + B^2 = \begin{pmatrix} a^2+b-1 & a-1 \\ b(a-1) & b \end{pmatrix} \text{ ....(ii)}$$

Equating (i) and (ii), we get $b = 4, a = 1$ 1 m

13. Using $C_1 \to C_1 + C_2 + C_3$ and taking $a^2 + a + 1$ common from C_1

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}, \text{ using } R_2 \to R_2 - R_1, \ R_3 \to R_3 - R_1$$ 1½ m

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & a(a-1) & (1-a)(1+a) \end{vmatrix} = (a^2+a+1)(1-a)^2\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & 1+a \end{vmatrix}$$ 1½ m

$$\left.\begin{aligned} &= (a^2+a+1)(1-a)^2(1+a+a^2) \\ &= [(1-a)(1+a+a^2)]^2 = (1-a^3)^2 \end{aligned}\right\}$$ 1 m

14. Let $x+a=t \Rightarrow dx=dt$ and $x=t-a \Rightarrow x-a=t-2a$ 1 m

$$\therefore\ I = \int \frac{\sin(t-2a)\,dt}{\sin t} = \int \frac{(\sin t\cos 2a-\cos t\cdot\sin 2a)\,dt}{\sin t}$$ 1 m

$$= \cos 2a\int dt-\sin 2a\int \cot t\,dt = \cos 2a\ t-\sin 2a\ .\log|\sin t|+c$$ 1 m

$$= \cos 2a\,(x+a)-\sin 2a\log|\sin(x+a)|+c$$ 1

OR

consider $\dfrac{x^2}{(x^2+4)(x^2+9)}$ · Let $x^2=t$ ½ m

$$\therefore\ \frac{t}{(t+4)(t+9)} = -\frac{4}{5}\ \frac{1}{t+4}+\frac{9}{5}\ \frac{1}{t+9}$$ 1 m

$$I = \int \frac{t\quad dt}{(t+4)(t+9)} = -\frac{4}{5}\int\frac{dt}{t+4}+\frac{9}{5}\int\frac{dt}{t+9}$$ ½ m

$$= \frac{-4}{5}\log|t+4|+\frac{9}{5}\log|t+9|+c$$ 1½ m

$$\therefore\ I = -\frac{4}{5}\log|x^2+4|+\frac{9}{5}\log|x^2+9|+c$$ ½ m

15. L H D at $x=1$: $\lim\limits_{x\to1^-}\left(\dfrac{x-1}{x-1}\right) = 1$

2 m

R H D at $x=1$, $\lim\limits_{x\to1^+}\dfrac{2-x-1}{x-1} = -1$

$\therefore$ f is not differentiable at $x=1$

L H D at $x=2$, $\lim\limits_{x\to2^-}\dfrac{2-x-0}{x-2} = -1$

2 m

R H D at $x=2$, $\lim\limits_{x\to2^+}\dfrac{-2+3x-x^2}{(x-2)} = \lim\limits_{x\to2^+} -\dfrac{(x-1)(x-2)}{(x-2)} = -1$

$\therefore$ f is diff. at $x=2$

16. $$\text{Communication Matrix } A = \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} \begin{array}{l} \text{Telephone} \\ \text{House calls} \\ \text{Letters} \end{array}$$

$$\text{Cost Matrix } B = \begin{array}{c} \begin{array}{ccc} \text{Tele} & \text{House calls} & \text{Letters} \end{array} \\ \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{City x} \\ \text{City y} \end{array}$$

$$\therefore \text{ Total cost Matrix } = \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} = \begin{pmatrix} 990000 \\ 2120000 \end{pmatrix}$$ 3 m

any relevant value 1 m

17. $$I = \int e^{2x} \sin(3x+1)\,dx = \left[\frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \int e^{2x} \cos(3x+1)\,dx\right]$$ 1½ m

$$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \cdot \frac{e^{2x}}{2} \cdot \cos(3x+1) - \frac{9}{4} \int e^{2x} \sin(3x+1)\,dx \Bigg]$$ 1 m

$$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) - \frac{9}{4} I$$ 1 m

$$\frac{13}{4} I = \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1)$$

$$I = \frac{4}{13}\left[\frac{e^{2x}}{2}\left(-\frac{3}{2}\cos(3x+1) + \sin(3x+1)\right)\right] + c$$ ½ m

18. $$9y^2 = x^3 \Rightarrow 18y \frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \text{slope of tangent} = \frac{x^2}{6y}$$

1+½ m

$$\therefore \text{ Slope of normal} = -\frac{6y}{x^2}$$

As the intercepts by normal on both axes are equal

$$\therefore \text{ Slope of normal} = \pm 1 \Rightarrow \frac{-6y}{x^2} = \pm 1 \Rightarrow y = \pm \frac{x^2}{6}$$ 1 m

$\therefore\ 9\left(\frac{x^4}{36}\right) = x^3 \Rightarrow x = 4$ and $y^2 = \frac{64}{9} \Rightarrow y = \pm 8/3$ 1 m

$\therefore$ The points are $\left(4, \frac{8}{3}\right), \left(4, -8/3\right)$ ½ m

19. $\frac{dy}{dx} = n\left(x + \sqrt{1+x^2}\right)^{n-1}\left[1 + \frac{x}{\sqrt{1+x^2}}\right] = \frac{n}{\sqrt{1+x^2}}\left[x + \sqrt{1+x^2}\right]^n = \frac{ny}{\sqrt{1+x^2}}$ 1½ m

$\therefore\ \sqrt{1+x^2}\,\frac{dy}{dx} = ny$............(i) ½ m

$\therefore\ \sqrt{1+x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{x}{\sqrt{1+x^2}} = n\frac{dy}{dx}$ 1 m

$\Rightarrow\ \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{1+x^2}\,\frac{dy}{dx} = n\cdot ny$ (from (i))

$= n^2 y$ 1 m

SECTION - C

20. Let $z = ax + by$, also $xy = c^2 \Rightarrow y = \frac{c^2}{x}$ ½ + ½ m

$\therefore\ z = ax + \frac{bc^2}{x}$

$\therefore\ \frac{dz}{dx} = a + bc^2\left(\frac{-1}{x^2}\right),\ \frac{dz}{dx} = 0 \Rightarrow bc^2 = ax^2$

or $x = \sqrt{\frac{b}{a}}\,c$ 1½ m

showing $\frac{d^2z}{ax^2}$ at $x = \sqrt{\frac{b}{a}}\, c > 0 \Rightarrow$ minima 1½ m

$y = \frac{c^2}{x} = \frac{c^2}{c}\sqrt{\frac{a}{b}} = c\sqrt{\frac{a}{b}}$ 1 m

$\therefore$ minimum $z = a\sqrt{\frac{b}{a}}\, c + bc\sqrt{\frac{a}{b}}\, c = 2\sqrt{ab}\, c$ 1 m

OR

$y = x^2 + 7x + 2\ ,\ 3x - y - 3 = 0$ ....(i)

$\therefore\ 3x - (x^2 + 7x + 2) - 3 = 0$ 1 m

Distance of (x, y) from (i)

$D = \left|\frac{3x - (x^2 + 7x + 2) - 3}{\sqrt{10}}\right|$ or $D = \left|\frac{(-x^2 - 4x - 5)}{\sqrt{10}}\right| = \left|\frac{(x+2)^2 + 1}{\sqrt{10}}\right|$ 2 m

$\frac{dD}{dx} = \frac{2}{\sqrt{10}}(x+2),\ \frac{dD}{dx} = 0$ at $x = -2$ 1 m

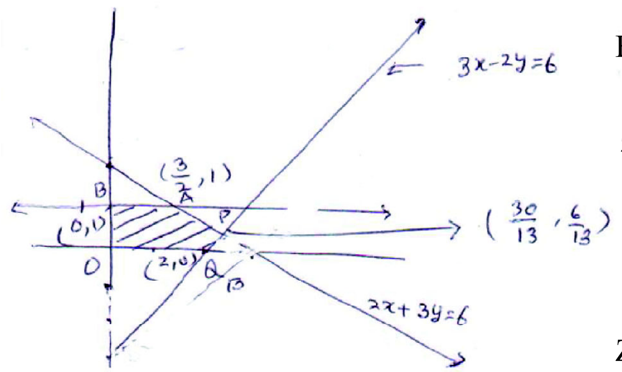
$\frac{d^2D}{dx^2} > 0 \Rightarrow$ minima 1 m

$\therefore$ D is minimum at $x = -2$

at $x = -2,\ y = -8$ 1 m

$\therefore$ The required pt. on the parabola is $(-2, -8)$

21. Figure 3 m



Feasible region is B A PQO 1 m

$z_B = 9,\ z_P = \frac{240}{13} + \frac{54}{13} = \frac{294}{13}$ 1 m

$= 22\frac{8}{13}$

$Z_Q = 16$

$\therefore$ Z is maximum at $\left(\frac{30}{13}, \frac{6}{13}\right)$ 1 m

and maximum value $= 22\frac{8}{13}$

22. Any line through $(1, -2, 3)$ with d. r's as 2, 3 – 6 is

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$ 1 ½ m

$\therefore\ x = 2\lambda + 1,\ y = 3\lambda - 2,\ z = -6\lambda + 3$ 1½ m

It lies on the plane $x - y + z = 5$

$\therefore\ 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$

$\Rightarrow\ \lambda = \frac{1}{7}$ 1 m

Reqd. point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$ 1 m

$\therefore$ Reqd distance $= \sqrt{\left(\frac{9}{7} - 1\right)^2 + \left(-\frac{11}{7} + 2\right)^2 + \left(\frac{15}{7} - 3\right)^2} = \frac{1}{7}\sqrt{4 + 9 + 36} = \frac{7}{7} = 1$ 1 m

23. Let $x_1, x_2, \in R$ such that $f(x_1) = f(x_2) \Rightarrow 4x_1^{\,2} + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ 1½+½ m

$\Rightarrow 4(x_1 - x_2)[x_1 + x_2 + 3] = 0 \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one – one

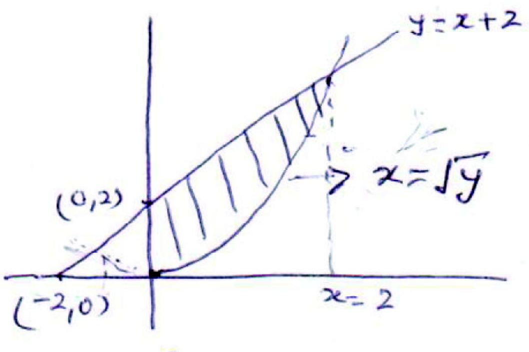
f is clearly onto and hence intertible 1 m

Let y be an arbitrary element of S

$f(x) = y = 4x^2 + 12x + 15 = (2x+3)^2 + 6$ 1 m

$\therefore f^{-1} : R \to S$ is given by $f^{-1}(y) = \left(\dfrac{\sqrt{y-6}-3}{2}\right)$ 2 m

24.



Correct Figure 1m

Points of intersection

$x^2 - x - 2 = 0$ 1½ m

$(x-2)(x+1) = 0$

$x = 2, -1$ (– 1 is rejected)

$\therefore$ Reqd. area $= \int_0^2 \{(x+2) - x^2\} dx$ 1½ m

$= \left[\dfrac{x^2}{2} + 2x - \dfrac{x^3}{3}\right]_0^2$

$= \left(2 + 4 - \dfrac{8}{3}\right) = \dfrac{10}{3}$ sq.units 2 m

25. $P(\text{Doublet}) = \dfrac{1}{6}$, $P(\text{not a doublet}) = 5/6$ 1 m

The random variate x can take values 0, 1, 2, 3, 4

x	0	1	2	3	4	
P(x)	$\left(\dfrac{5}{6}\right)^4$	$4\,\dfrac{1}{6}\left(\dfrac{5}{6}\right)^3$	$6\left(\dfrac{1}{6}\right)^2\left(\dfrac{5}{6}\right)^2$	$4\left(\dfrac{1}{6}\right)^3\dfrac{5}{6}$	$\left(\dfrac{1}{6}\right)^4$	
	$\dfrac{625}{1296}$	$\dfrac{500}{1296}$	$\dfrac{150}{1296}$	$\dfrac{20}{1296}$	$\dfrac{1}{1296}$	2½ m

$$\text{Mean} = \sum x\, P(x) = \frac{500+300+60+4}{1296} = \frac{864}{1296} = \frac{2}{3}$$ 1 m

$$\sum x^2\, P(x) = \frac{500+600+180+16}{1296} = \frac{1296}{1296} = 1$$ 1 m

$$\therefore \text{ Variance } = 1-\left(\frac{2}{3}\right)^2 = \frac{5}{9}$$ ½ m

26. $$\frac{dy}{dx} = \frac{2x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)}{y - x\cos\left(\frac{y}{x}\right)} = \frac{2\sin\left(\frac{y}{x}\right) - \frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{y}{x} - \cos\left(\frac{y}{x}\right)}$$ 1 m

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ 1 m

$$\therefore\ v + x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v} \Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v^2}{v - \cos v}$$ 1 m

$$\Rightarrow \frac{v - \cos v}{-2\sin v + v^2} = \frac{-dx}{x} \text{ or } \frac{1}{2}\left[\frac{2v - 2\cos v}{-2\sin v + v^2}\right]dv = -\frac{dx}{x}$$ 1 m

$$\Rightarrow \frac{1}{2}\log\left|v^2 - 2\sin v\right| = -\log x + \log c$$

$$\text{or } \log\left|\sqrt{v^2 - 2\sin v}\right| = \log c - \log x$$ 1 m

$$\sqrt{v^2 - 2\sin v} = \frac{c}{x}$$

$$\text{or } x\sqrt{\frac{y^2}{x^2} - 2\sin\frac{y}{x}} = c$$ ½ m

$$y^2 - 2x^2\sin\left(\frac{y}{x}\right) = c'$$ ½ m

OR

$$\left(\sqrt{(1+x^2)(1+y^2)}\right) dx + xy\,dy = 0$$

$$\Rightarrow \quad \frac{y}{\sqrt{1+y^2}}\, dy + \frac{\sqrt{1+x^2}}{x}\, dx = 0$$ 1 m

$$\frac{1}{2}\int \frac{2y}{\sqrt{1+y^2}}\, dy + \int \frac{\sqrt{1+x^2}}{x}\, dx = 0$$

$$\sqrt{1+y^2} + \int \frac{(1+x^2)}{x\sqrt{1+x^2}}\, dx = c$$ 1½ m

$$I_1 = \int \frac{1}{x\sqrt{1+x^2}}\, dx + \int \frac{x}{\sqrt{1+x^2}}\, dx = I_2 + \sqrt{1+x^2}$$ 1 m

For I_2, Let $x = \frac{1}{t}$, $dx = \frac{-1}{t^2}\, dt$ 1 m

$$I_2 = \int \frac{-1}{t^2 \cdot \frac{1}{t}\sqrt{1+\frac{1}{t^2}}}\, dt = -\int \frac{dt}{\sqrt{t^2+1}} = -\log\left[t + \sqrt{1+t^2}\right]$$ 1 m

$$= -\log\left[\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right] = -\log\left[\frac{1+\sqrt{1+x^2}}{x}\right]$$

½ m

$\therefore$ The solution is $\sqrt{1+x^2} + \sqrt{1+y^2} - \log\left(\frac{1+\sqrt{1+x^2}}{2}\right) = c$

QUESTION PAPER CODE 65/3/F

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. Reqd. area $= \left| \vec{a} \times \vec{b} \right|$ ½ m

$\therefore \left| 12\hat{i} - 4\hat{j} + 8\hat{k} \right| = \sqrt{144+16+64} = \sqrt{224}$ or $4\sqrt{14}$ sq. units ½ m

2. Getting x – intercept $= \dfrac{5}{2}$, y – intercept $= 5$, z – intercept $= -5$ ½ m

$\therefore$ Their sum $= \dfrac{5}{2}$ ½ m

3. $\vec{a} + \vec{b} = 6\hat{i} + \hat{k}$ ½ m

$\therefore$ Reqd. unit vector $= \dfrac{6}{\sqrt{37}}\hat{i} + \dfrac{1}{\sqrt{37}}\hat{k}$ ½ m

4. Degree = Order = 2 any one correct ½ m

$\therefore$ Degree + order = 4 ½ m

5. $2^y\, dy = dx \Rightarrow \dfrac{2^y}{\log 2} = x + c$ ½+½ m

6. co – factor of $a_{21} = 3$ 1 m

SECTION - B

7. $9y^2 = x^3 \Rightarrow 18y\dfrac{dy}{dx} = 3x^2 \Rightarrow \dfrac{dy}{dx} = \text{slope of tangent} = \dfrac{x^2}{6y}$

$\therefore$ Slope of normal $= -\dfrac{6y}{x^2}$ 1+½ m

As the intercepts by normal on both axes are equal

$\therefore$ Slope of normal $= \pm 1 \Rightarrow \dfrac{-6y}{x^2} = \pm 1 \Rightarrow y = \pm\dfrac{x^2}{6}$ 1 m

$\therefore 9\left(\dfrac{x^4}{36}\right) = x^3 \Rightarrow x = 4$ and $y^2 = \dfrac{64}{9} \Rightarrow y = \pm 8/3$ 1 m

$\therefore$ The points are $\left(4, \dfrac{8}{3}\right), \left(4, -8/3\right)$ ½ m

8. $\dfrac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left[1+\dfrac{x}{\sqrt{1+x^2}}\right] = \dfrac{n}{\sqrt{1+x^2}}\left[x+\sqrt{1+x^2}\right]^n = \dfrac{ny}{\sqrt{1+x^2}}$ 1½ m

$\therefore \sqrt{1+x^2}\dfrac{dy}{dx} = ny$...........(i) ½ m

$\therefore \sqrt{1+x^2}\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx}\cdot\dfrac{x}{\sqrt{1+x^2}} = n\dfrac{dy}{dx}$ 1 m

$\Rightarrow \left(1+x^2\right)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = n\sqrt{1+x^2}\dfrac{dy}{dx} = n\cdot ny$ (from (i))

$= n^2 y$ 1 m

9. $\text{L H D at } x = 1 : \lim_{x\to1^-} \left(\frac{x-1}{x-1}\right) = 1$

$\text{R H D at } x = 1,\ \lim_{x\to1^+} \frac{2-x-1}{x-1} = -1$ 2 m

$\therefore$ f is not differentiable at $x = 1$

$\text{L H D at } x = 2,\ \lim_{x\to2^-} \frac{2-x-0}{x-2} = -1$

$\text{R H D at } x = 2,\ \lim_{x\to2^+} \frac{-2+3x-x^2}{(x-2)} = \lim_{x\to2^+} -\frac{(x-1)(x-2)}{(x-2)} = -1$ 2 m

$\therefore$ f is diff. at $x = 2$

10. Communication Matrix $A = \begin{array}{c|l} \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} & \begin{array}{l}\text{Telephone} \\ \text{House calls} \\ \text{Letters}\end{array} \end{array}$

Cost Matrix $B = \begin{array}{cl} \begin{array}{ccc} \text{Tele} & \text{House calls} & \text{Letters} \end{array} & \\ \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} & \begin{array}{l}\text{City x} \\ \text{City y}\end{array} \end{array}$

$\therefore$ Total cost Matrix $= \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} = \begin{pmatrix} 990000 \\ 2120000 \end{pmatrix}$ 3 m

any relevant value 1 m

11. $I = \int e^{2x} \sin(3x+1)\,dx = \left[\frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \int e^{2x} \cos(3x+1)\,dx\right]$ 1½ m

$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \cdot \frac{e^{2x}}{2} \cdot \cos(3x+1) - \frac{9}{4} \int e^{2x} \sin(3x+1)\,dx \Big]$ 1 m

$$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) - \frac{9}{4} I$$ 1 m

$$\frac{13}{4} I = \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1)$$

$$I = \frac{4}{13}\left[\frac{e^{2x}}{2}\left(-\frac{3}{2}\cos(3x+1) + \sin(3x+1)\right)\right] + c$$ ½ m

12. Writing given integral as 1 m

$$I = \int_{-\pi/2}^{0} \frac{\cos x}{1+e^x}\,dx + \int_{0}^{\pi/2} \frac{\cos x}{1+e^x}\,dx$$ Let $x = -t$, $dx = -dt$ 1½ m

when $x = -\frac{\pi}{2}$, $t = \pi/2$

$x = 0$, $t = 0$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{e^t \cos t}{1+e^t}\,dt + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx = \int_0^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx$$ 1½ m

$$I = \int_0^{\frac{\pi}{2}} \frac{(1+e^x)\cos x\,dx}{(1+e^x)} = \int_0^{\frac{\pi}{2}} \cos x\,dx = (\sin x)_0^{\frac{\pi}{2}} = 1$$ 1 m

13. Let B_1, B_2, B_3 be the events that the bolts produced by machines E_1, E_2, E_3 and A be the event that the selected bulb is defective ½ m

$$\therefore P(B_1) = \frac{1}{2},\ P(B_2) = P(B_3) = \frac{1}{4}$$

1½ m

$$P\left(A/B_1\right) = \frac{1}{25},\ P\left(A/B_2\right) = \frac{1}{25},\ P\left(A/B_3\right) = \frac{1}{20}$$

$$P(A) = \sum_{c=1}^{3} P(B_c)\,P\left(A/B_c\right) = \frac{1}{2}\times\frac{1}{25} + \frac{1}{4}\times\frac{1}{20} + \frac{1}{4}\times\frac{1}{25}$$ 1+½ m

$$= \frac{17}{400}$$ ½ m

OR

$$\therefore\ P(x=3) = \frac{1}{6} \times \frac{1}{5} \times 2 = \frac{1}{15},\quad P(x=4) = \frac{2}{6} \times \frac{1}{5} \times 2 = \frac{2}{15}$$

Simlarly $P(x=5) = \frac{3}{15}$, $P(x=6) = \frac{4}{15}$, $P(x=7) = \frac{5}{15}$

Prob. distribution is

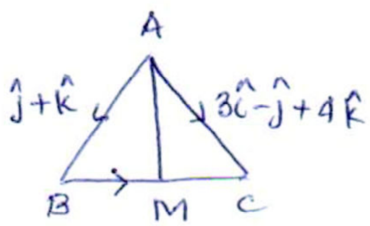
x :	3	4	5	6	7
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$
x . P (x) :	$\frac{3}{15}$	$\frac{8}{15}$	$\frac{15}{15}$	$\frac{24}{15}$	$\frac{35}{15}$
x^2 P (x) :	$\frac{9}{15}$	$\frac{32}{15}$	$\frac{75}{15}$	$\frac{144}{15}$	$\frac{245}{15}$

2 m

$$\text{Mean} = \sum x_i \cdot P(x_i) = \frac{85}{15} = \frac{17}{3}$$ 1 m

$$\text{Variance} = \sum x_i^2\ P(x_i) - (\text{Mean})^2 = \frac{101}{3} - \frac{289}{9} = \frac{14}{9}$$ 1 m

14.



$$\overrightarrow{BC} = \left(3\hat{i} - \hat{j} + 4\hat{k}\right) - \left(\hat{j} + \hat{k}\right) = 3\hat{i} - 2\hat{j} + 3\hat{k}$$ 1½ m

$$\therefore\ \overrightarrow{BM} = \frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}$$ 1 m

$$AM = \left|\hat{j} + \hat{k} + \frac{3\hat{i} - 2\hat{j} + 3\hat{k}}{2}\right| = \left|\frac{3\hat{i} + 5\hat{k}}{2}\right| = \frac{\sqrt{34}}{2}$$ 1½ m

15. Any plane through given point is $a(x-3)+b(y-6)+c(z-4)=0$..........(i) 1 m

with $a+5b+4c=0$ ....(A) ½ m

(i) passes through $(3, 2, 0)$ $\Rightarrow -4b-4c=0$ or $b+c=0$ ....(B) ½ m

From (A) and (B) $a+b+(4b+4c) = 0 \Rightarrow a=-b$ 1 m

$$\left.\begin{aligned}&\therefore\ a=-b=c\\&\therefore\ \text{Required eqn. of plane is } x-y+z-1=0\end{aligned}\right\}$$ 1 m

16. $\text{LHS} = \tan^{-1}\left(\dfrac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta}$ 2 m

$\therefore\ \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta} = \tan^{-1}\left(\dfrac{2}{\sin\theta}\right)$ 1 m

$\Rightarrow\ \cot\theta = 1$ or $\theta = \pi/4$ 1 m

OR

The given equation can be written

$\left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)+\left(\tan^{-1}4-\tan^{-1}3\right)+.....+\tan^{-1}(n+1)-\tan^{-1}n = \tan^{-1}\theta$ 2 m

$\Rightarrow\ \tan^{-1}(n+1)-\tan^{-1}1 = \tan^{-1}\theta$ 1 m

$\Rightarrow\ \tan^{-1}\dfrac{n+1-1}{1+(n+1)} = \tan^{-1}\theta \Rightarrow \theta = \dfrac{n}{n+2}$ 1 m

17. Getting $A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$ 1 m

$4A - 3I = \begin{bmatrix} 8-3 & -4 \\ -4 & 8-3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} = A^2$ 1 m

$A^2 = 4A - 3I$ ....(i)

Multiply both sides by A^{-1} ½ m

$A = 4I - 3A^{-1}$ or $A^{-1} = \frac{1}{3}(4I - A) = \frac{1}{3}\begin{pmatrix} 4-2 & 0+1 \\ 0+1 & 4-2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ 1½ m

OR

$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a^2+b & a-1 \\ b(a-1) & b+1 \end{bmatrix}$ 1½ m

$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a) - 2(2+b) & 4 \end{pmatrix}$ .... (i) 1½ m

$A^2 + B^2 = \begin{pmatrix} a^2+b-1 & a-1 \\ b(a-1) & b \end{pmatrix}$ ....(ii)

Equating (i) and (ii), we get $b = 4, a = 1$ 1 m

18. Using $C_1 \to C_1 + C_2 + C_3$ and taking $a^2 + a + 1$ common from C_1

$\Delta = (a^2 + a + 1)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}$, using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ 1½ m

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & a(a-1) & (1-a)(1+a) \end{vmatrix} = (a^2+a+1)(1-a)^2 \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & 1+a \end{vmatrix}$$ 1½ m

$$= (a^2+a+1)(1-a)^2(1+a+a^2)$$

$$= [(1-a)(1+a+a^2)]^2 = (1-a^3)^2$$ 1 m

19. Let $x+a=t \Rightarrow dx = dt$ and $x = t-a \Rightarrow x-a = t-2a$ 1 m

$$\therefore\ I = \int \frac{\sin(t-2a)\,dt}{\sin t} = \int \frac{(\sin t \cos 2a - \cos t \cdot \sin 2a)\,dt}{\sin t}$$ 1 m

$$= \cos 2a \int dt - \sin 2a \int \cot t\, dt = \cos 2a\ t - \sin 2a\,.\log|\sin t| + c$$ 1 m

$$= \cos 2a\,(x+a) - \sin 2a \log|\sin(x+a)| + c$$ 1

OR

consider $\dfrac{x^2}{(x^2+4)(x^2+9)}$ · Let $x^2 = t$ ½ m

$$\therefore\ \frac{t}{(t+4)(t+9)} = -\frac{4}{5}\frac{1}{t+4} + \frac{9}{5}\frac{1}{t+9}$$ 1 m

$$I = \int \frac{t\ \ dt}{(t+4)(t+9)} = -\frac{4}{5}\int\frac{dt}{t+4} + \frac{9}{5}\int\frac{dt}{t+9}$$ ½ m

$$= \frac{-4}{5}\log|t+4| + \frac{9}{5}\log|t+9| + c$$ 1½ m

$$\therefore\ I = -\frac{4}{5}\log|x^2+4| + \frac{9}{5}\log|x^2+9| + c$$ ½ m

SECTION - C

20. $$\frac{dy}{dx} = \frac{2x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)}{y - x\cos\left(\frac{y}{x}\right)} = \frac{2\sin\left(\frac{y}{x}\right) - \frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{y}{x} - \cos\left(\frac{y}{x}\right)}$$ 1 m

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$\therefore\ v + x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v} \Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v^2}{v - \cos v}$ 1 m

$\Rightarrow \frac{v - \cos v}{-2\sin v + v^2} = \frac{-dx}{x}$ or $\frac{1}{2}\left[\frac{2v - 2\cos v}{-2\sin v + v^2}\right]dv = -\frac{dx}{x}$ 1 m

$$\left.\begin{aligned}&\Rightarrow \frac{1}{2}\log\left|v^2 - 2\sin v\right| = -\log x + \log c\\ &\text{or } \log\left|\sqrt{v^2 - 2\sin v}\right| = \log c - \log x\end{aligned}\right\}$$ 1 m

$\sqrt{v^2 - 2\sin v} = \frac{c}{x}$

or $x\sqrt{\frac{y^2}{x^2} - 2\sin\frac{y}{x}} = c$ ½ m

$y^2 - 2x^2\sin\left(\frac{y}{x}\right) = c'$ ½ m

OR

$\left(\sqrt{(1+x^2)(1+y^2)}\right)dx + xy\,dy = 0$

$\Rightarrow \frac{y}{\sqrt{1+y^2}}\,dy + \frac{\sqrt{1+x^2}}{x}\,dx = 0$ 1 m

$\frac{1}{2}\int\frac{2y}{\sqrt{1+y^2}}\,dy + \int\frac{\sqrt{1+x^2}}{x}\,dx = 0$

$\sqrt{1+y^2} + \int\frac{(1+x^2)}{x\sqrt{1+x^2}}\,dx = c$ 1½ m

$$I_1 = \int \frac{1}{x\sqrt{1+x^2}}\,dx + \int \frac{x}{\sqrt{1+x^2}}\,dx = I_2 + \sqrt{1+x^2}$$ 1 m

For I_2, Let $x = \frac{1}{t}$, $dx = \frac{-1}{t^2}\,dt$ 1 m

$$I_2 = \int \frac{-1}{t^2 \cdot \frac{1}{t}\sqrt{1+\frac{1}{t^2}}}\,dt = -\int \frac{dt}{\sqrt{t^2+1}} = -\log\left[t+\sqrt{1+t^2}\right]$$ 1 m

$$= -\log\left[\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right] = -\log\left[\frac{1+\sqrt{1+x^2}}{x}\right]$$

$\therefore$ The solution is $\sqrt{1+x^2}+\sqrt{1+y^2} - \log\left(\frac{1+\sqrt{1+x^2}}{2}\right) = c$ ½ m

21. $P(\text{Doublet}) = \frac{1}{6}$, $P(\text{not a doublet}) = \frac{5}{6}$

The random variate x can take values 0, 1, 2, 3, 4 1 m

x	0	1	2	3	4	
P(x)	$\left(\frac{5}{6}\right)^4$	$4\,\frac{1}{6}\left(\frac{5}{6}\right)^3$	$6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$	$4\left(\frac{1}{6}\right)^3\frac{5}{6}$	$\left(\frac{1}{6}\right)^4$	
	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$	2½ m

$$\text{Mean} = \sum x\,P(x) = \frac{500+300+60+4}{1296} = \frac{864}{1296} = \frac{2}{3}$$ 1 m

$$\sum x^2\,P(x) = \frac{500+600+180+16}{1296} = \frac{1296}{1296} = 1$$ 1 m

$$\therefore \text{Variance} = 1-\left(\frac{2}{3}\right)^2 = \frac{5}{9}$$ ½ m

22. Let $x_1, x_2, \in R$ such that $f(x_1) = f(x_2) \Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ 1½+½ m

$\Rightarrow 4(x_1 - x_2)[x_1 + x_2 + 3] = 0 \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one – one

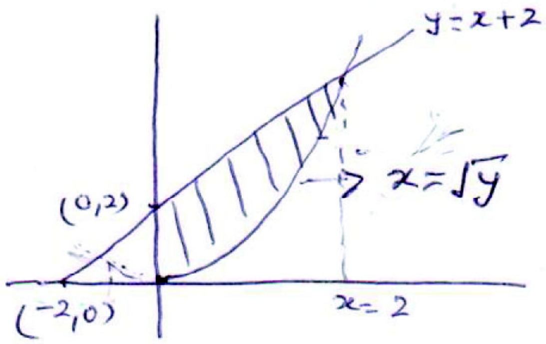
f is clearly onto and hence intertible 1 m

Let y be an arbitrary element of S

$f(x) = y = 4x^2 + 12x + 15 = (2x+3)^2 + 6$ 1 m

$\therefore\ f^{-1} : R \to S$ is given by $f^{-1}(y) = \left(\dfrac{\sqrt{y-6}-3}{2}\right)$ 2 m

23.



Correct Figure 1m

Points of intersection

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$ 1½ m

$x = 2, -1$ (– 1 is rejected)

$\therefore$ Reqd. area $= \int_0^2 \{(x+2) - x^2\}dx$ 1½ m

$= \left[\dfrac{x^2}{2} + 2x - \dfrac{x^3}{3}\right]_0^2$

$= \left(2 + 4 - \dfrac{8}{3}\right) = \dfrac{10}{3}$ sq.units 2 m

24.

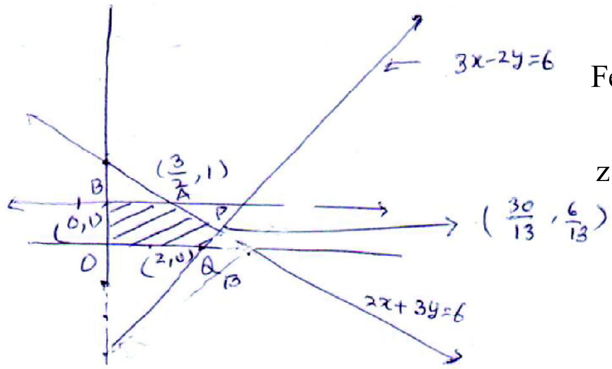


Figure 3 m

Feasible region is B A PQO 1 m

$z_B = 9,\ z_P = \dfrac{240}{13} + \dfrac{54}{13} = \dfrac{294}{13}$

$= 22\dfrac{8}{13}$ 1 m

$Z_Q = 16$

$\therefore$ Z is maximum at $\left(\dfrac{30}{13}, \dfrac{6}{13}\right)$

and maximum value $= 22\dfrac{8}{13}$ 1 m

25. Any line through $(1,-2,3)$ with d. r's as 2, 3 – 6 is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$ 1 ½ m

$\therefore\ x = 2\lambda+1,\ y = 3\lambda-2,\ z = -6\lambda+3$ 1½ m

It lies on the plane $x-y+z=5$

$\therefore\ 2\lambda+1-3\lambda+2-6\lambda+3=5$

$\Rightarrow\ \lambda = \frac{1}{7}$ 1 m

Reqd. point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$ 1 m

$\therefore$ Reqd distance $= \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(-\frac{11}{7}+2\right)^2 + \left(\frac{15}{7}-3\right)^2} = \frac{1}{7}\sqrt{4+9+36} = \frac{7}{7} = 1$ 1 m

26. Let $z = ax + by$, also $xy = c^2 \Rightarrow y = \frac{c^2}{x}$ ½ + ½ m

$\therefore\ z = ax + \frac{bc^2}{x}$

$\therefore\ \frac{dz}{dx} = a + bc^2\left(\frac{-1}{x^2}\right),\ \frac{dz}{dx} = 0 \Rightarrow bc^2 = ax^2$

or $x = \sqrt{\frac{b}{a}}\,c$ 1½ m

showing $\frac{d^2z}{ax^2}$ at $x = \sqrt{\frac{b}{a}}\,c > 0 \Rightarrow$ minima 1½ m

$y = \frac{c^2}{x} = \frac{c^2}{c}\sqrt{\frac{a}{b}} = c\sqrt{\frac{a}{b}}$ 1 m

$\therefore$ minimum $z = a\sqrt{\frac{b}{a}}\,c + bc\sqrt{\frac{a}{b}}\,c = 2\sqrt{ab}\,c$ 1 m

OR

$y = x^2 + 7x + 2$, $3x - y - 3 = 0$ ....(i)

$\therefore\ 3x - \left(x^2 + 7x + 2\right) - 3 = 0$ 1 m

Distance of (x, y) from (i)

$D = \left|\frac{3x - \left(x^2 + 7x + 2\right) - 3}{\sqrt{10}}\right|$ or $D = \left|\frac{\left(-x^2 - 4x - 5\right)}{\sqrt{10}}\right| = \left|\frac{(x+2)^2 + 1}{\sqrt{10}}\right|$ 2 m

$\frac{dD}{dx} = \frac{2}{\sqrt{10}}(x+2)$, $\frac{dD}{dx} = 0$ at $x = -2$ 1 m

$\frac{d^2D}{dx^2} > 0 \Rightarrow$ minima 1 m

$\therefore$ D is minimum at $x = -2$

at $x = -2$, $y = -8$ 1 m

$\therefore$ The required pt. on the parabola is $(-2, -8)$