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Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65/1/B, 65/2/B, 65/3/B

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/B

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $x = 2\ ,\ \ y = 9$ (½ for correct x or y)

$\therefore\ \ x + y = 11$ ½ m

2. order 3 , or degree 1 ½ m

$\therefore$ Degree + order = 4 ½ m

3. $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x}$ (Standard form) ½ m

I.F. = $\log x$ ½ m

4. $\vec{a} \cdot \vec{b} = 0 \Rightarrow x = -6$ ½ m

$y = \pm\sqrt{40}$ or $\pm 2\sqrt{10}$ ½ m

5. $a^2 \sin^2 \alpha + a^2 \sin^2 \beta + a^2 \sin^2 \gamma$ ½ m

$= 2a^2$ ½ m

6. using $\sin\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$ ½ m

$\Rightarrow \theta = 0^\circ$ ½ m

SECTION - B

7. $[15000 \quad 15000] \begin{bmatrix} \frac{2}{100} \\ \frac{x}{100} \end{bmatrix} = [1800]$ 2 m

$\Rightarrow \quad 300 + 150x = 1800$ 1 m

$\Rightarrow \quad x = 10\%$

yes : compassionate or any other relevant value 1 m

8. $\cot^{-1}(x+1) = \sin^{-1}\dfrac{1}{\sqrt{1+(x+1)^2}}$ 1½ m

and $\tan^{-1}x = \cos^{-1}\dfrac{1}{\sqrt{1+x^2}}$ 1½ m

$$\therefore \quad \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$\Rightarrow \quad 1 + x^2 + 2x + 1 = 1 + x^2 \Rightarrow \quad x = -\dfrac{1}{2}$ 1 m

OR

$$2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$$

$= \quad 2\tan^{-1}\dfrac{3}{4} - \tan^{-1}\dfrac{17}{31}$ 1 m

$= \quad \tan^{-1}\dfrac{24}{7} - \tan^{-1}\dfrac{17}{31}$ 1 m

$= \quad \tan^{-1}1 = \dfrac{\pi}{4}$ 1+1 m

9. $C_1 \to C_1 + C_2 + C_3$,

$(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$ 1 m

$$R_2 \to R_2 - R_1, \ R_3 \to R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0 \quad (\because\ a+b+c \neq 0)$$ 2 m

$$\Rightarrow \quad -a^2 - b^2 - c^2 + ab + bc + ca = 0$$ ½ m

$$\Rightarrow \quad -\frac{1}{2}\,[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$ ½ m

$$\Rightarrow \quad a = b = c$$

10. $$\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$ 1 m

$R_2 \to R_2 - 2R_1$,

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$R_2 \to R_2 - 3R_3$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$R_1 \to R_1 + R_2$, $R_3 \to R_3 - 2R_2$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & +7 \end{pmatrix} \cdot A$$ (2 marks for all operations)

$$\therefore \quad A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix}$$ 1 m

11. $f(x) = x - \left| x - x^2 \right| = \left| x - x(1-x) \right| = \begin{cases} 2x - x^2 & , \quad -1 \le x < 0 \\ 0 & , \quad x = 0 \\ x^2 & , \quad 0 < x \le 1 \end{cases}$ 1 m

$f(x)$ being a polynomial is continuous on $[-1, 0] \cup [0, 1]$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} (2x - x^2) = 0$ ½ m

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} x^2 = 0$ ½ m

Also, $f(0) = 0$

$\because \lim\limits_{x \to 0^-} f(x) = f(0) = \lim\limits_{x \to 0^+} f(x)$ 1 m

$\Rightarrow$ There is no point of discontinuity on $[-1, 1]$ 1 m

12. $\dfrac{y}{x} = [\log x - \log(a + bx)]$ ½ m

$\Rightarrow \dfrac{x\dfrac{dy}{dx} - y}{x^2} = \dfrac{1}{x} - \dfrac{b}{a + bx}$ 1 m

$\Rightarrow x\dfrac{dy}{dx} - y = \dfrac{ax}{a + bx}$ .... (i) 1 m

Differentiating again,

$x\dfrac{d^2y}{dx^2} = \dfrac{a^2}{(a + bx)^2}$ 1 m

$x^3 \cdot \dfrac{d^2y}{dx^2} = \left(\dfrac{ax}{a + bx}\right)^2 = \left(x\dfrac{dy}{dx} - y\right)^2$ (using (i)) ½ m

13. $u = \sec^{-1}\left(\dfrac{1}{2x^2 - 1}\right) = 2\cos^{-1}x \Rightarrow \dfrac{du}{dx} \; \dfrac{-2}{\sqrt{1 - x^2}}$ 1½ m

$v = \sqrt{1 - x^2} \Rightarrow \dfrac{dv}{dx} = \dfrac{-x}{\sqrt{1 - x^2}}$ 1 m

$\left.\dfrac{dv}{dx}\right|_{x = \frac{1}{2}} = \dfrac{2}{x} = 4$ 1½ m

14. Let $I = \int_0^{\pi/2} \frac{5\sin x + 3\cos x}{\sin x + \cos x}\, dx$ .... (i)

$\Rightarrow \quad I = \int_0^{\pi/2} \frac{5\cos x + 3\sin x}{\cos x + \sin x}\, dx$ .... (ii) $\left(\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \right)$ 1½ m

Adding (i) and (ii) 1+1 m

$2I = 8\int_0^{\pi/2} 1 \cdot dx = 4\pi$ ½ m

$\Rightarrow \quad I = 2\pi$

OR

put $\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t\, dt$ 1 m

$= \int e^t \left(\log t + \frac{1}{t^2} \right) dt$

$= \int e^t \left[\left(\log t - \frac{1}{t} \right) + \left(\frac{1}{t} + \frac{1}{t^2} \right) \right] dt$ 1½ m

$= e^t \left(\log t - \frac{1}{t} \right) + c$ 1 m

$= x \left[\log(\log x) - \frac{1}{\log x} \right] + c$ ½ m

15. $I = \int \frac{x\cos x}{\cos x + x\sin x}\, dx$ 1 m

put $\cos x + x\sin x = t$

$\Rightarrow \quad x\cos x\, dx = dt$ 1 m

$= \int \frac{dt}{t}$ 1 m

$= \log |\cos x + x\sin x| + c$ 1 m

16. $\int \frac{x^4\,dx}{(x-1)(x^2+1)} = \int \left[(x+1) + \frac{1}{(x-1)(x^2+1)}\right] dx$ 1 m

(using partial fractions)

$= \int (x+1)\,dx + \frac{1}{2}\int \frac{dx}{(x-1)} - \frac{1}{2}\int \frac{x+1}{x^2+1}\,dx$ 1½ m

$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1} x + c$ 1½ m

17. $\left.\begin{aligned}\overrightarrow{AB} &= -2\hat{i} - 5\hat{k}\\ \overrightarrow{AC} &= \hat{i} - 2\hat{j} - \hat{k}\end{aligned}\right\}$ 1 m

$\overrightarrow{AB} \times \overrightarrow{AC} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ 1 m

$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{165}$ ½ m

$\hat{n} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|}$ 1 m

$= \frac{(-10\hat{i} - 7\hat{j} + 4\hat{k})}{\sqrt{165}}$ or $\frac{10\hat{i} + 7\hat{j} - 4\hat{k}}{\sqrt{165}}$ ½ m

18. $\left.\begin{aligned}\vec{a}_1 &= -\hat{i},\ \vec{b}_1 = \hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k}\\ \vec{a}_2 &= -2\hat{j} + \hat{k},\ \vec{b}_2 = \hat{i} + \hat{j} + \frac{1}{6}\hat{k}\end{aligned}\right\}$ 1 m

$\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} + \hat{k}$ ½ m

$\vec{b}_1 \times \vec{b}_2 = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k}$ ½ m

$\left|\vec{b}_1 \times \vec{b}_2\right| = \frac{7}{12}$ 1 m

S.D. $= \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = 2$ 1 m

OR

Foot of perpendicular are (0, b, c) & (a, 0, c) 1 m

Equ. of required plane

$$\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$$ 2 m

$\Rightarrow$ $bcx + acy - abz = 0$ 1 m

19. $p(x = 2) = 9 . P(x = 3)$ 1 m

$\Rightarrow$ ${}^3C_2\, p^2\, q = 9 \cdot {}^3C_3\, p^3 \cdot q^0$ 1 m

$\Rightarrow$ $3\,p^2\,(1 - p) = 9\,p^3$ 1 m

$\Rightarrow$ $p = \frac{1}{4}$ 1 m

OR

Let H_1 be the event that red ball is drawn

H_2 be the event that black ball is drawn

E be the event that both balls are red

$P(H_1) = \frac{3}{8}, \quad P(H_2) = \frac{5}{8}$ 1 m

$P(E/H_1) = \frac{5_{C_2}}{10_{C_2}} = \frac{2}{9}, \quad P(E/H_2) = \frac{3_{C_2}}{10_{C_2}} = \frac{1}{15}$ 1 m

$P(E) = P(H_1)\ P(E/H_1) + P(H_2) \cdot P(E/H_2)$ 1 m

$= \frac{3}{8} \cdot \frac{2}{9} + \frac{5}{8} \cdot \frac{1}{15} = \frac{1}{8}$ 1 m

20.

*	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

4 m

$\forall \ a \in \{0, 1, 2, 3, 4, 5, 6\}$

$a * 0 = a = 0 * a \Rightarrow$ 0 is identity 1 m

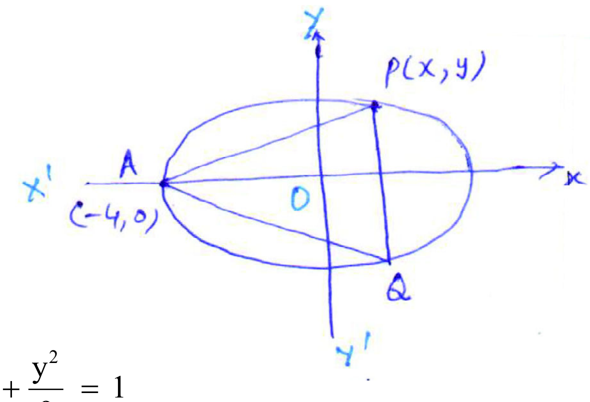
$\forall \ a \in \{1, 2, 3, 4, 5, 6\}$

$a * b = 0 = b * a$

$\Rightarrow \quad a * (7 - a) = 0 = (7 - a) * a$

$\Rightarrow \quad (7 - a)$ is inverse of a 1 m

21. $A = y(x + 4)$



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Let $z = A^2 = \frac{9}{16}(16 - x^2)(x + 4)^2 \Rightarrow y^2 = \frac{9}{16}(16 - x^2)$ .... (i) 1 m

$$= \frac{9}{16}(4 - x)(4 + x)^3$$ 1 m

$$\frac{dz}{dx} = \frac{9}{16}(4+x)^2\,(8-4x)$$ 1 m

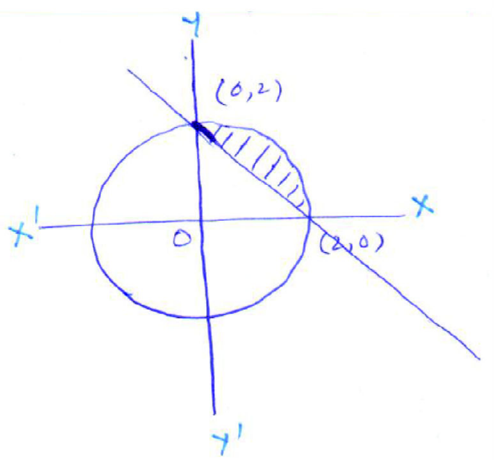
$$\frac{dz}{dx} = 0 \;\Rightarrow\; x = 2$$ 1 m

$$\frac{d^2z}{dx^2} = -\frac{9}{4}(4+x)^2 + \frac{9}{8}(4+x)\,(8-4x)$$

$$\left.\frac{d^2z}{dx^2}\right|_{x=2} < 0$$ 1 m

$\therefore$ Maximum value of A = $9\sqrt{3}$ sq. units 1 m

22.



1 m

Required Area

$$= \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx$$ 2 m

$$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$ 1+1 m

$$= (\pi - 2) \text{ sq. units}$$ 1 m

23. $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

put $y = v\,x \Rightarrow \frac{dy}{dx} = v + x\,\frac{dv}{dx}$ 1 m

$\Rightarrow \frac{1+v^2}{v^3} = -\frac{dx}{x}$ 1 m

Integrating both sides

$-\frac{1}{2\,v^2} + \log v = -\log x + c$ 1 m

$\Rightarrow -\frac{x^2}{2y^2} + \log y = c$

when $x = 1,\; y = 1 \Rightarrow c = -\frac{1}{2}$ 1 m

$\Rightarrow \log y = \frac{x^2 - y^2}{2y^2}$ ½ m

when $x = x_0,\; y = e \Rightarrow x_0 = \sqrt{3}\,e$ 1½ m

OR

$I\;F = e^{\int \tan x\,dx} = e^{\log \sec x} = \sec x$ 1 m

$\therefore \frac{d}{dx}(y \cdot \sec x) = 3x^2 \sec x + x^3 \sec x \tan x$ 1 m

$\Rightarrow y \sec x = \int 3x^2 \sec x \cdot dx + x^3 \sec x - \int 3x^2 \cdot \sec x\,dx + c$ 2 m

$\Rightarrow y = x^3 + c\cos x$

when $x = \frac{\pi}{3},\; y = 0$; we get $c = \frac{-2\pi^3}{27}$ 1 m

$\therefore y = x^3 - \frac{2\pi^3}{27}\cos x$ 1 m

24. Equation of line is $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$ 1 m

Equation of plane is

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ 1 & 0 & -2 \\ 2 & -3 & -1 \end{vmatrix} = 0$$ 1 m

$\Rightarrow$ $2x + y + z - 7 = 0$ .... (i) 1 m

general point on given line $(2\lambda + 3, -3\lambda + 4, 5\lambda + 1)$ lies on (i) 1 m

$\therefore$ $2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) - 7 = 0 \Rightarrow \lambda = -\frac{2}{3}$ 1 m

$\therefore$ Point of intersection $\left(\frac{5}{3}, 6, -\frac{7}{3}\right)$ 1 m

25. Let H_1 : be the event 1, 2 appears

H_2 : be the event 3, 4, 5, 6 appears 1 m

E_3 : be the event that head appears

$P(H_1) = \frac{2}{6} = \frac{1}{3}$, $P(H_2) = \frac{4}{6} = \frac{2}{3}$ 1 m

$P(E/H_1) = \frac{3}{8}$ $P(E/H_2) = \frac{1}{2}$ 1 m

$$P(H_2/E) = \frac{P(H_2) \cdot P(E/H_2)}{P(H_1) \cdot P(E/H_1) + P(H_2)\ P(E/H_2)}$$ 1 m

$= \frac{8}{11}$ 2 m

OR

Let H_1 : be the event that 4 occurs

H_2 : be the event that 4 does not occurs 1 m

E : be the event that man reports 4 occurs

on a throw of dice

$P(H_1) = \frac{1}{6}$, $P(H_2) = \frac{5}{6}$ 1 m

$P(E/H_1) = \frac{3}{5}$ $P(E/H_2) = 1 - \frac{3}{5} = \frac{2}{5}$ 1 m

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)}$$ 1 m

$$= \frac{3}{13}$$ 2 m

26. Let us consider the man invested on x electronic and y manually operated machines

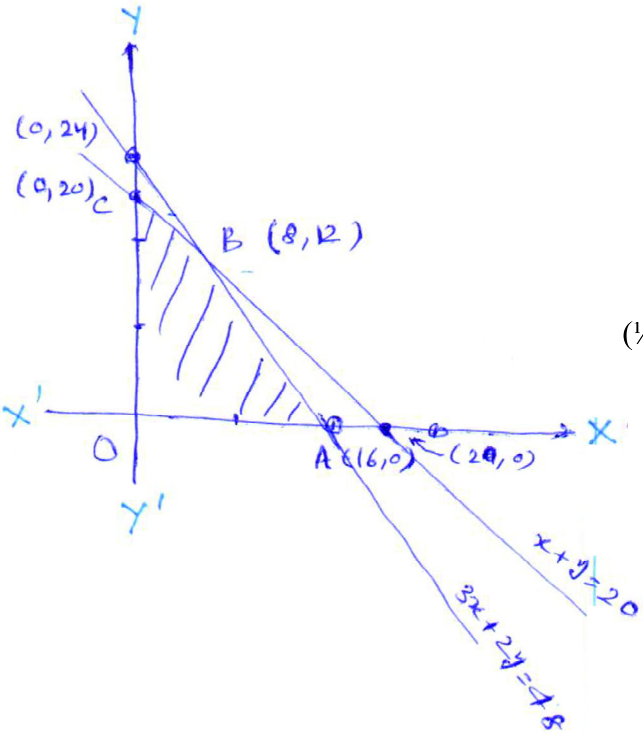
Maximise P = 220 x + 180 y .... (i) 1 m

subject to

$x + y \leq 20$

$3600\,x + 2400\,y \leq 57600 \quad \Rightarrow \quad 3x + 2y \leq 48$ 1½ m

$x, y \geq 0$



(1 mark for plotting each line) = 2 m

(½ to find the vertices of feasible region)

$P\big|_{A\,(16,\,0)} = 3520$ Rs.

$P\big|_{B\,(8,\,12)} = 3920$ Rs.

$P\big|_{C\,(0,\,20)} = 3600$ Rs.

Maximum profit is Rs. 3920 at x = 8, y = 12 1 m

QUESTION PAPER CODE 65/2/B

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\vec{a} \cdot \vec{b} = 0 \Rightarrow x = -6$ ½ m

$y = \pm\sqrt{40}$ or $\pm 2\sqrt{10}$ ½ m

2. $a^2 \sin^2 \alpha + a^2 \sin^2 \beta + a^2 \sin^2 \gamma$ ½ m

$= 2a^2$ ½ m

3. using $\sin\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$ ½ m

$\Rightarrow \theta = 0^{\circ}$ ½ m

4. $x = 2,\ y = 9$ (½ for correct x or y)

$\therefore\ x + y = 11$ ½ m

5. order 3 , or degree 1 ½ m

$\therefore$ Degree + order = 4 ½ m

6. $\dfrac{dy}{dx} + \dfrac{y}{x \log x} = \dfrac{2}{x}$ (Standard form) ½ m

I.F. = $\log x$ ½ m

SECTION - B

7. $\dfrac{y}{x} = [\log x - \log(a + bx)]$ ½ m

$\Rightarrow \dfrac{x\dfrac{dy}{dx} - y}{x^2} = \dfrac{1}{x} - \dfrac{b}{a + bx}$ 1 m

$\Rightarrow \quad x\frac{dy}{dx} - y = \frac{ax}{a+bx}$ .... (i) 1 m

Differentiating again,

$$x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2}$$ 1 m

$x^3 \cdot \frac{d^2y}{dx^2} = \left(\frac{ax}{a+bx}\right)^2 = \left(x\frac{dy}{dx} - y\right)^2$ (using (i)) ½ m

8. $u = \sec^{-1}\left(\frac{1}{2x^2-1}\right) = 2\cos^{-1}x \Rightarrow \frac{du}{dx}\ \frac{-2}{\sqrt{1-x^2}}$ 1½ m

$v = \sqrt{1-x^2} \Rightarrow \frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}}$ 1 m

$\left.\frac{dv}{dx}\right|_{x=\frac{1}{2}} = \frac{2}{x} = 4$ 1½ m

9. Let $I = \int_0^{\pi/2} \frac{5\sin x + 3\cos x}{\sin x + \cos x}\,dx$ .... (i)

$\Rightarrow \quad I = \int_0^{\pi/2} \frac{5\cos x + 3\sin x}{\cos x + \sin x}\,dx$ .... (ii) $\left(\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$ 1½ m

Adding (i) and (ii) 1+1 m

$$2I = 8\int_0^{\pi/2} 1 \cdot dx = 4\pi$$ ½ m

$$\Rightarrow \quad I = 2\pi$$

OR

put $\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t\,dt$ 1 m

$= \int e^t \left(\log t + \frac{1}{t^2}\right) dt$

$$= \int e^t \left[\left(\log t - \frac{1}{t}\right) + \left(\frac{1}{t} + \frac{1}{t^2}\right)\right] dt$$ 1½ m

$$= e^t \left(\log t - \frac{1}{t}\right) + c$$ 1 m

$$= x\left[\log(\log x) - \frac{1}{\log x}\right] + c$$ ½ m

10. $$[15000 \quad 15000]\begin{bmatrix} \frac{2}{100} \\ \frac{x}{100} \end{bmatrix} = [1800]$$ 2 m

$\Rightarrow \quad 300 + 150x = 1800$ 1 m

$\Rightarrow \quad x = 10\%$

yes : compassionate or any other relevant value 1 m

11. $$\cot^{-1}(x+1) = \sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}$$ 1½ m

and $$\tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$$ 1½ m

$$\therefore \quad \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad 1 + x^2 + 2x + 1 = 1 + x^2 \Rightarrow x = -\frac{1}{2}$$ 1 m

OR

$$2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$$

$$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$$ 1 m

$= \quad \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$ 1 m

$= \quad \tan^{-1} 1 = \frac{\pi}{4}$ 1+1 m

12. $C_1 \to C_1 + C_2 + C_3$,

$$(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$ 1 m

$R_2 \to R_2 - R_1 ,\ R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0 \quad (\because\ a+b+c \neq 0)$$ 2 m

$\Rightarrow \quad -a^2 - b^2 - c^2 + ab + bc + ca = 0$ ½ m

$\Rightarrow \quad -\frac{1}{2}\,[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ ½ m

$\Rightarrow \quad a = b = c$

13. $$\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$ 1 m

$R_2 \to R_2 - 2R_1$,

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$R_2 \to R_2 - 3R_3$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$R_1 \to R_1 + R_2\,,\quad R_3 \to R_3 - 2R_2$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & +7 \end{pmatrix} \cdot A$$ (2 marks for all operations)

$$\therefore \quad A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix}$$ 1 m

14. $f(x) = x - \left| x - x^2 \right| = \left| x - x(1-x) \right| = \begin{cases} 2x - x^2 & , \quad -1 \le x < 0 \\ 0 & , \quad x = 0 \\ x^2 & , \quad 0 < x \le 1 \end{cases}$ 1 m

f(x) being a polynomial is continuous on $[-1, 0] \cup [0, 1]$

$\underset{x \to 0^-}{\text{lt}}\ f(x) = \underset{x \to 0^-}{\text{lt}}\ (2x - x^2) = 0$ ½ m

$\underset{x \to 0^+}{\text{lt}}\ f(x) = \underset{x \to 0^+}{\text{lt}}\ x^2 = 0$ ½ m

Also, $f(0) = 0$

$\because \lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$ 1 m

$\Rightarrow$ There is no point of discontinuity on $[-1, 1]$ 1 m

15. $\left.\begin{aligned} \vec{a}_1 &= -\hat{i}\,,\ \vec{b}_1 = \hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k} \\ \vec{a}_2 &= -2\hat{j} + \hat{k}\,,\ \vec{b}_2 = \hat{i} + \hat{j} + \frac{1}{6}\hat{k} \end{aligned}\right\}$ 1 m

$\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} + \hat{k}$ ½ m

$\vec{b}_1 \times \vec{b}_2 = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k}$ ½ m

$\left| \vec{b}_1 \times \vec{b}_2 \right| = \frac{7}{12}$ 1 m

S.D. $= \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{\left| \vec{b}_1 \times \vec{b}_2 \right|} \right| = 2$ 1 m

OR

Foot of perpendicular are (0, b, c) & (a, 0, c) 1 m

Equ. of required plane

$$\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$$ 2 m

$\Rightarrow \quad bcx + acy - abz = 0$ 1 m

16. $p\,(x = 2) = 9 \,.\, P(x = 3)$ 1 m

$\Rightarrow \ {}^3C_2\, p^2\, q = 9 \cdot {}^3C_3\, p^3 \cdot q^0$ 1 m

$\Rightarrow \ 3\,p^2\,(1 - p) = 9\,p^3$ 1 m

$\Rightarrow \quad p = \frac{1}{4}$ 1 m

OR

Let H_1 be the event that red ball is drawn

H_2 be the event that black ball is drawn

E be the event that both balls are red

$P(H_1) = \frac{3}{8}, \quad P(H_2) = \frac{5}{8}$ 1 m

$P(E/H_1) = \frac{5_{C_2}}{10_{C_2}} = \frac{2}{9}, \quad P(E/H_2) = \frac{3_{C_2}}{10_{C_2}} = \frac{1}{15}$ 1 m

$P(E) = P(H_1)\ P(E/H_1) + P(H_2) \cdot P(E/H_2)$ 1 m

$$= \frac{3}{8} \cdot \frac{2}{9} + \frac{5}{8} \cdot \frac{1}{15} = \frac{1}{8}$$ 1 m

17. $I = \int \frac{x \cos x}{\cos x + x \sin x} dx$ 1 m

put $\cos x + x \sin x = t$

$\Rightarrow \quad x \cos x \, dx = dt$ 1 m

$= \int \frac{dt}{t}$ 1 m

$= \log |\cos x + x \sin x| + c$ 1 m

18. $\int \frac{x^4 \, dx}{(x-1)(x^2+1)} = \int \left[(x+1) + \frac{1}{(x-1)(x^2+1)} \right] dx$ 1 m

(using partial fractions)

$= \int (x+1) \, dx + \frac{1}{2} \int \frac{dx}{(x-1)} - \frac{1}{2} \int \frac{x+1}{x^2+1} dx$ 1½ m

$= \frac{x^2}{2} + x + \frac{1}{2} \log |x-1| - \frac{1}{4} \log (x^2+1) - \frac{1}{2} \tan^{-1} x + c$ 1½ m

19. $\left.\begin{aligned} \overrightarrow{AB} &= -2\hat{i} - 5\hat{k} \\ \overrightarrow{AC} &= \hat{i} - 2\hat{j} - \hat{k} \end{aligned}\right\}$ 1 m

$\overrightarrow{AB} \times \overrightarrow{AC} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ 1 m

$\left| \overrightarrow{AB} \times \overrightarrow{AC} \right| = \sqrt{165}$ ½ m

$\hat{n} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{\left| \overrightarrow{AB} \times \overrightarrow{AC} \right|}$ 1 m

$= \frac{(-10\hat{i} - 7\hat{j} + 4\hat{k})}{\sqrt{165}}$ or $\frac{10\hat{i} + 7\hat{j} - 4\hat{k}}{\sqrt{165}}$ ½ m

SECTION - C

20. $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

put $y = v\,x \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$\Rightarrow \frac{1+v^2}{v^3} = -\frac{dx}{x}$ 1 m

Integrating both sides

$-\frac{1}{2\,v^2} + \log v = -\log x + c$ 1 m

$\Rightarrow -\frac{x^2}{2y^2} + \log y = c$

when $x = 1,\ y = 1 \Rightarrow c = -\frac{1}{2}$ 1 m

$\Rightarrow \log y = \frac{x^2 - y^2}{2y^2}$ ½ m

when $x = x_0,\ y = e \Rightarrow x_0 = \sqrt{3}\,e$ 1½ m

OR

$I\ F = e^{\int \tan x\,dx} = e^{\log \sec x} = \sec x$ 1 m

$\therefore \frac{d}{dx}(y \cdot \sec x) = 3x^2 \sec x + x^3 \sec x \tan x$ 1 m

$\Rightarrow y \sec x = \int 3x^2 \sec x \cdot dx + x^3 \sec x - \int 3x^2 \cdot \sec x\,dx + c$ 2 m

$\Rightarrow y = x^3 + c \cos x$

when $x = \frac{\pi}{3},\ y = 0;$ we get $c = \frac{-2\pi^3}{27}$ 1 m

$\therefore \quad y = x^3 - \frac{2\pi^3}{27}\cos x$ 1 m

21. Equation of line is $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$ 1 m

Equation of plane is

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ 1 & 0 & -2 \\ 2 & -3 & -1 \end{vmatrix} = 0$$ 1 m

$\Rightarrow \quad 2x + y + z - 7 = 0$ .... (i) 1 m

general point on given line $(2\lambda + 3, -3\lambda + 4, 5\lambda + 1)$ lies on (i) 1 m

$\therefore \quad 2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) - 7 = 0 \Rightarrow \lambda = -\frac{2}{3}$ 1 m

$\therefore$ Point of intersection $\left(\frac{5}{3}, 6, -\frac{7}{3}\right)$ 1 m

22.

*	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

4 m

$\forall \; a \in \{0, 1, 2, 3, 4, 5, 6\}$

$a * 0 = a = 0 * a \Rightarrow 0$ is identity 1 m

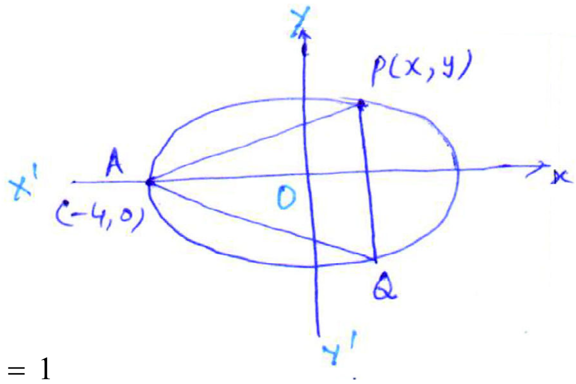
$\forall\ a \in \{1, 2, 3, 4, 5, 6\}$

$a * b = 0 = b * a$

$\Rightarrow \quad a * (7 - a) = 0 = (7 - a) * a$

$\Rightarrow \quad (7 - a)$ is inverse of a 1 m

23. $A = y(x + 4)$



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Let $z = A^2 = \frac{9}{16}(16 - x^2)(x + 4)^2 \quad \Rightarrow \quad y^2 = \frac{9}{16}(16 - x^2)$ .... (i) 1 m

$$= \frac{9}{16}(4 - x)(4 + x)^3$$ 1 m

$$\frac{dz}{dx} = \frac{9}{16}(4 + x)^2 (8 - 4x)$$ 1 m

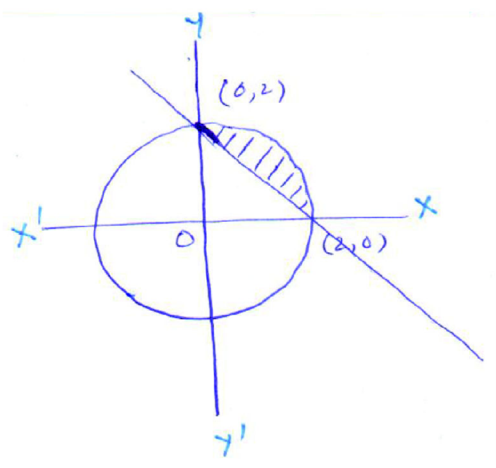
$$\frac{dz}{dx} = 0 \Rightarrow x = 2$$ 1 m

$$\frac{d^2z}{dx^2} = -\frac{9}{4}(4 + x)^2 + \frac{9}{8}(4 + x)(8 - 4x)$$

$$\left.\frac{d^2z}{dx^2}\right|_{x=2} < 0$$ 1 m

$\therefore$ Maximum value of $A = 9\sqrt{3}$ sq. units 1 m

24.



1 m

Required Area

$$= \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx$$ 2 m

$$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$ 1+1 m

$$= (\pi - 2) \text{ sq. units}$$ 1 m

25. Let us consider the man invested on x electronic and y manually operated machines

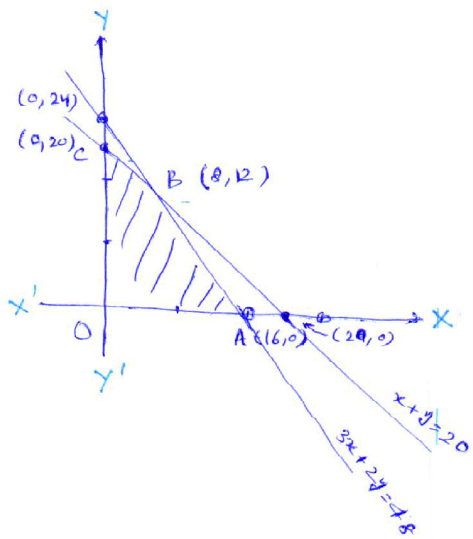
Maximise P = 220 x + 180 y .... (i) 1 m

subject to

$x + y \leq 20$

$3600\,x + 2400\,y \leq 57600 \quad \Rightarrow \quad 3x + 2y \leq 48$ 1½ m

$x, y \geq 0$



(1 mark for plotting each line) = 2 m

(½ to find the vertices of feasible region)

$P\big|_{A(16,0)} = 3520$ Rs.

$P\big|_{B(8,12)} = 3920$ Rs.

$P\big|_{C(0,20)} = 3600$ Rs.

Maximum profit is Rs. 3920 at x = 8, y = 12 1 m

26. Let H_1 : be the event 1, 2 appears

H_2 : be the event 3, 4, 5, 6 appears 1 m

E_3 : be the event that head appears

$P(H_1) = \frac{2}{6} = \frac{1}{3}, \quad P(H_2) = \frac{4}{6} = \frac{2}{3}$ 1 m

$P(E/H_1) = \frac{3}{8} \quad P(E/H_2) = \frac{1}{2}$ 1 m

$$P(H_2/E) = \frac{P(H_2) \cdot P(E/H_2)}{P(H_1) \cdot P(E/H_1) + P(H_2)\ P(E/H_2)}$$ 1 m

$$= \frac{8}{11}$$ 2 m

OR

Let H_1 : be the event that 4 occurs

H_2 : be the event that 4 does not occurs 1 m

E : be the event that man reports 4 occurs

on a throw of dice

$P(H_1) = \frac{1}{6}, \quad P(H_2) = \frac{5}{6}$ 1 m

$P(E/H_1) = \frac{3}{5} \quad P(E/H_2) = 1 - \frac{3}{5} = \frac{2}{5}$ 1 m

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)}$$ 1 m

$$= \frac{3}{13}$$ 2 m

QUESTION PAPER CODE 65/3/B

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x}$ (Standard form) ½ m

I.F. = $\log x$ ½ m

2. $x = 2$, $y = 9$ (½ for correct x or y)

$\therefore$ $x + y = 11$ ½ m

3. order 3 , or degree 1 ½ m

$\therefore$ Degree + order = 4 ½ m

4. using $\sin \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$ ½ m

$\Rightarrow \theta = 0^{o}$ ½ m

5. $\vec{a} \cdot \vec{b} = 0 \Rightarrow x = -6$ ½ m

$y = \pm \sqrt{40}$ or $\pm 2\sqrt{10}$ ½ m

6. $a^2 \sin^2 \alpha + a^2 \sin^2 \beta + a^2 \sin^2 \gamma$ ½ m

$= 2 a^2$ ½ m

SECTION - B

7. $\left.\begin{array}{l} \overrightarrow{AB} = -2\hat{i} - 5\hat{k} \\ \overrightarrow{AC} = \hat{i} - 2\hat{j} - \hat{k} \end{array}\right\}$ 1 m

$\overrightarrow{AB} \times \overrightarrow{AC} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ 1 m

$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{165}$ ½ m

$$\hat{n} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|}$$ 1 m

$$= \frac{\left(-10\hat{i} - 7\hat{j} + 4\hat{k}\right)}{\sqrt{165}} \text{ or } \frac{10\hat{i} + 7\hat{j} - 4\hat{k}}{\sqrt{165}}$$ ½ m

8. $\left.\begin{array}{l} \vec{a}_1 = -\hat{i}, \ \vec{b}_1 = \hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k} \\ \vec{a}_2 = -2\hat{j} + \hat{k}, \ \vec{b}_2 = \hat{i} + \hat{j} + \frac{1}{6}\hat{k} \end{array}\right\}$ 1 m

$\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} + \hat{k}$ ½ m

$\vec{b}_1 \times \vec{b}_2 = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k}$ ½ m

$\left|\vec{b}_1 \times \vec{b}_2\right| = \frac{7}{12}$ 1 m

$\text{S.D.} = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = 2$ 1 m

OR

Foot of perpendicular are (0, b, c) & (a, 0, c) 1 m

Equ. of required plane

$$\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$$ 2 m

$\Rightarrow \quad bcx + acy - abz = 0$ 1 m

9. $p(x = 2) = 9 . P(x = 3)$ 1 m

$\Rightarrow \ {}^3C_2\, p^2\, q = 9 \cdot {}^3C_3\, p^3 \cdot q^0$ 1 m

$\Rightarrow \ 3p^2(1 - p) = 9p^3$ 1 m

$\Rightarrow \quad p = \frac{1}{4}$ 1 m

OR

Let H_1 be the event that red ball is drawn

H_2 be the event that black ball is drawn

E be the event that both balls are red

$$P(H_1) = \frac{3}{8}, \quad P(H_2) = \frac{5}{8}$$ 1 m

$$P(E/H_1) = \frac{5_{C_2}}{10_{C_2}} = \frac{2}{9}, \quad P(E/H_2) = \frac{3_{C_2}}{10_{C_2}} = \frac{1}{15}$$ 1 m

$$P(E) = P(H_1)\ P(E/H_1) + P(H_2) \cdot P(E/H_2)$$ 1 m

$$= \frac{3}{8} \cdot \frac{2}{9} + \frac{5}{8} \cdot \frac{1}{15} = \frac{1}{8}$$ 1 m

10. $\frac{y}{x} = [\log x - \log (a + b x)]$ ½ m

$$\Rightarrow \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{b}{a + b x}$$ 1 m

$$\Rightarrow x\frac{dy}{dx} - y = \frac{ax}{a + b x} \quad \text{.................................. (i)}$$ 1 m

Differentiating again,

$$x\frac{d^2y}{dx^2} = \frac{a^2}{(a + b x)^2}$$ 1 m

$$x^3 \cdot \frac{d^2y}{dx^2} = \left(\frac{ax}{a + b x}\right)^2 = \left(x\frac{dy}{dx} - y\right)^2 \quad \text{(using (i))}$$ ½ m

11. $u = \sec^{-1}\left(\frac{1}{2x^2-1}\right) = 2\cos^{-1}x \Rightarrow \frac{du}{dx}\ \frac{-2}{\sqrt{1-x^2}}$ 1½ m

$v = \sqrt{1-x^2} \Rightarrow \frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}}$ 1 m

$\left.\frac{dv}{dx}\right|_{x=\frac{1}{2}} = \frac{2}{x} = 4$ 1½ m

12. Let $I = \int_0^{\pi/2} \frac{5\sin x + 3\cos x}{\sin x + \cos x}\, dx$ .... (i)

$\Rightarrow \quad I = \int_0^{\pi/2} \frac{5\cos x + 3\sin x}{\cos x + \sin x}\, dx$ .... (ii) $\left(\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$ 1½ m

Adding (i) and (ii) 1+1 m

$2I = 8\int_0^{\pi/2} 1 \cdot dx = 4\pi$ ½ m

$\Rightarrow \quad I = 2\pi$

OR

put $\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t\, dt$ 1 m

$= \int e^t \left(\log t + \frac{1}{t^2}\right) dt$

$= \int e^t \left[\left(\log t - \frac{1}{t}\right) + \left(\frac{1}{t} + \frac{1}{t^2}\right)\right] dt$ 1½ m

$= e^t\left(\log t - \frac{1}{t}\right) + c$ 1 m

$= x\left[\log(\log x) - \frac{1}{\log x}\right] + c$ ½ m

13. $I = \int \frac{x \cos x}{\cos x + x \sin x} dx$ 1 m

put $\cos x + x \sin x = t$

$\Rightarrow \quad x \cos x \, dx = dt$ 1 m

$= \int \frac{dt}{t}$ 1 m

$= \log | \cos x + x \sin x | + c$ 1 m

14. $\int \frac{x^4 \, dx}{(x-1)(x^2+1)} = \int \left[(x+1) + \frac{1}{(x-1)(x^2+1)} \right] dx$ 1 m

(using partial fractions)

$= \int (x+1) dx + \frac{1}{2} \int \frac{dx}{(x-1)} - \frac{1}{2} \int \frac{x+1}{x^2+1} dx$ 1½ m

$= \frac{x^2}{2} + x + \frac{1}{2} \log | x-1 | - \frac{1}{4} \log (x^2+1) - \frac{1}{2} \tan^{-1} x + c$ 1½ m

15. $[15000 \quad 15000] \begin{bmatrix} \frac{2}{100} \\ \frac{x}{100} \end{bmatrix} = [1800]$ 2 m

$\Rightarrow \quad 300 + 150x = 1800$ 1 m

$\Rightarrow \quad x = 10\%$

yes : compassionate or any other relevant value 1 m

16. $\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$ 1½ m

and $\tan^{-1} x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$ 1½ m

$$\therefore \quad \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$\Rightarrow \quad 1 + x^2 + 2x + 1 = 1 + x^2 \Rightarrow \quad x = -\frac{1}{2}$ 1 m

OR

$$2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$$

$= \quad 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$ 1 m

$= \quad \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$ 1 m

$= \quad \tan^{-1} 1 = \frac{\pi}{4}$ 1+1 m

17. $C_1 \to C_1 + C_2 + C_3$,

$(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$ 1 m

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0 \quad (\because\ a+b+c \neq 0)$ 2 m

$\Rightarrow \quad -a^2 - b^2 - c^2 + ab + bc + ca = 0$ ½ m

$\Rightarrow \quad -\frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] = 0$ ½ m

$\Rightarrow \quad a = b = c$

18. $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$ 1 m

$R_2 \to R_2 - 2R_1$,

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$R_2 \to R_2 - 3R_3$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$R_1 \to R_1 + R_2$, $R_3 \to R_3 - 2R_2$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & +7 \end{pmatrix} \cdot A$ (2 marks for all operations)

$\therefore \quad A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix}$ 1 m

19. $f(x) = x - |x - x^2| = |x - x(1-x)| = \begin{cases} 2x - x^2 & , \quad -1 \le x < 0 \\ 0 & , \quad x = 0 \\ x^2 & , \quad 0 < x \le 1 \end{cases}$ 1 m

f(x) being a polynomial is continuous on $[-1, 0] \cup [0, 1]$

$\underset{x \to 0^-}{\text{lt}} f(x) = \underset{x \to 0^-}{\text{lt}} (2x - x^2) = 0$ ½ m

$\underset{x \to 0^+}{\text{lt}} f(x) = \underset{x \to 0^+}{\text{lt}} x^2 = 0$ ½ m

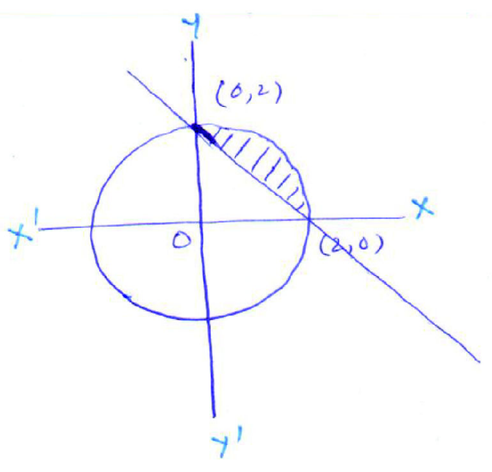
Also, $f(0) = 0$

$\because \lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$ 1 m

$\Rightarrow$ There is no point of discontinuity on $[-1, 1]$ 1 m

SECTION - C

20.



1 m

Required Area

$$= \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx$$ 2 m

$$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$ 1+1 m

$$= (\pi - 2) \text{ sq. units}$$ 1 m

21. $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$$\Rightarrow \frac{1+v^2}{v^3} = -\frac{dx}{x}$$ 1 m

Integrating both sides

$$-\frac{1}{2v^2} + \log v = -\log x + c$$ 1 m

$$\Rightarrow \quad -\frac{x^2}{2y^2} + \log y = c$$

when $x = 1,\ y = 1 \quad \Rightarrow \quad c = -\frac{1}{2}$ 1 m

$$\Rightarrow \quad \log y = \frac{x^2 - y^2}{2y^2}$$ ½ m

when $x = x_0,\ y = e \quad \Rightarrow \quad x_0 = \sqrt{3}\, e$ 1½ m

OR

$I\ F = e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$ 1 m

$\therefore \quad \frac{d}{dx}(y \cdot \sec x) = 3x^2 \sec x + x^3 \sec x \tan x$ 1 m

$\Rightarrow \quad y \sec x = \int 3x^2 \sec x \cdot dx + x^3 \sec x - \int 3x^2 \cdot \sec x\, dx + c$ 2 m

$\Rightarrow \quad y = x^3 + c \cos x$

when $x = \frac{\pi}{3},\ y = 0$; we get $c = \frac{-2\pi^3}{27}$ 1 m

$\therefore \quad y = x^3 - \frac{2\pi^3}{27} \cos x$ 1 m

22. Let us consider the man invested on x

electronic and y manually operated machines

Maximise $P = 220\,x + 180\,y$ .... (i) 1 m

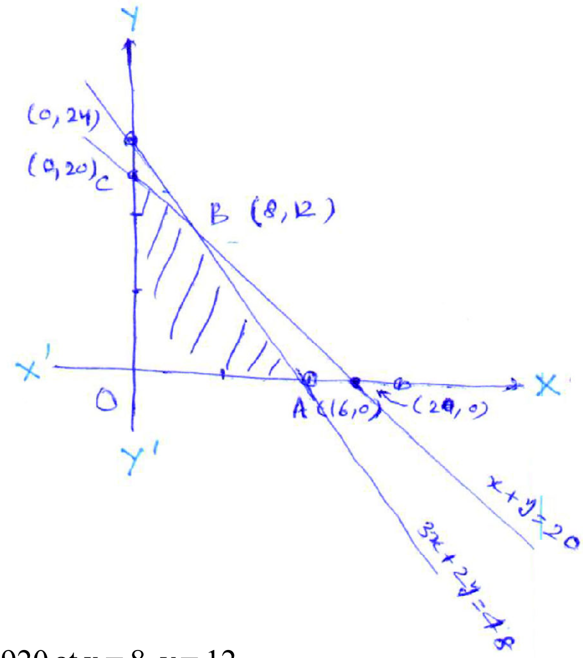
subject to

$x + y \leq 20$

$3600\,x + 2400\,y \leq 57600 \quad \Rightarrow \quad 3x + 2y \leq 48$ 1½ m

$x, y \geq 0$

(1 mark for



plotting each line) = 2 m

(½ to find the vertices of feasible region)

$P|_{A(16,0)} = 3520$ Rs.

$P|_{B(8,12)} = 3920$ Rs.

$P|_{C(0,20)} = 3600$ Rs.

Maximum profit is Rs. 3920 at x = 8, y = 12 1 m

23. Equation of line is $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$ 1 m

Equation of plane is

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ 1 & 0 & -2 \\ 2 & -3 & -1 \end{vmatrix} = 0$$ 1 m

$\Rightarrow \quad 2x + y + z - 7 = 0$ .... (i) 1 m

general point on given line $(2\lambda + 3, -3\lambda + 4, 5\lambda + 1)$ lies on (i) 1 m

$\therefore \quad 2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) - 7 = 0 \Rightarrow \lambda = -\frac{2}{3}$ 1 m

$\therefore$ Point of intersection $\left(\frac{5}{3}, 6, -\frac{7}{3}\right)$ 1 m

24. Let H_1 : be the event 1, 2 appears

H_2 : be the event 3, 4, 5, 6 appears 1 m

E_3 : be the event that head appears

$P(H_1) = \frac{2}{6} = \frac{1}{3}$, $P(H_2) = \frac{4}{6} = \frac{2}{3}$ 1 m

$P(E/H_1) = \frac{3}{8}$ $P(E/H_2) = \frac{1}{2}$ 1 m

$$P(H_2/E) = \frac{P(H_2) \cdot P(E/H_2)}{P(H_1) \cdot P(E/H_1) + P(H_2)\ P(E/H_2)}$$ 1 m

$$= \frac{8}{11}$$ 2 m

OR

Let H_1 : be the event that 4 occurs

H_2 : be the event that 4 does not occurs 1 m

E : be the event that man reports 4 occurs

on a throw of dice

$P(H_1) = \frac{1}{6}$, $P(H_2) = \frac{5}{6}$ 1 m

$P(E/H_1) = \frac{3}{5}$ $P(E/H_2) = 1 - \frac{3}{5} = \frac{2}{5}$ 1 m

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)}$$ 1 m

$$= \frac{3}{13}$$ 2 m

25.

*	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

4 m

$\forall\ a \in \{0, 1, 2, 3, 4, 5, 6\}$

$a * 0 = a = 0 * a \Rightarrow 0$ is identity 1 m

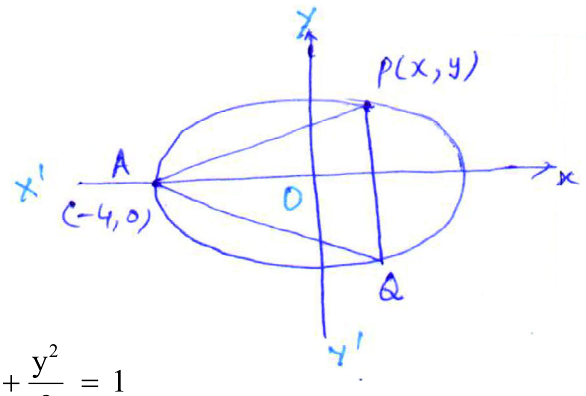
$\forall\ a \in \{1, 2, 3, 4, 5, 6\}$

$a * b = 0 = b * a$

$\Rightarrow \quad a * (7 - a) = 0 = (7 - a) * a$

$\Rightarrow \quad (7 - a)$ is inverse of a 1 m

26. $A = y\,(x + 4)$



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Let $z = A^2 = \frac{9}{16}(16 - x^2)(x + 4)^2 \quad \Rightarrow \quad y^2 = \frac{9}{16}(16 - x^2)$ .... (i) 1 m

$$= \frac{9}{16}(4 - x)(4 + x)^3$$ 1 m

$$\frac{dz}{dx} = \frac{9}{16}(4 + x)^2 (8 - 4x)$$ 1 m

$$\frac{dz}{dx} = 0 \Rightarrow x = 2$$ 1 m

$$\frac{d^2z}{dx^2} = -\frac{9}{4}(4 + x)^2 + \frac{9}{8}(4 + x)(8 - 4x)$$

$$\left.\frac{d^2z}{dx^2}\right|_{x=2} < 0$$ 1 m

$\therefore$ Maximum value of A = $9\sqrt{3}$ sq. units 1 m