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Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65/1/A, 65/2/A, 65/3/A

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/A

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. getting $|A| = 1$ ½ m

$|A^n| = 1$ ½ m

2. Order 2 or degree = 1 ½ m

sum = 3 ½ m

3. Writing $\int \frac{y}{\sqrt{1+y^2}} dy = -\int \frac{x\,dx}{\sqrt{1+x^2}}$ ½ m

Getting $\sqrt{1+y^2} + \sqrt{1+x^2} = c$ ½ m

4. $\overrightarrow{OB} = \frac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ ½ m

$\overrightarrow{OC} = 2\vec{b} - \vec{a}$ ½ m

5. Vector Perpendicular to $\vec{a}$ and $\vec{b}$ $= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

[Finding or using] ½ m

Required Vector $= \hat{i} - 11\hat{j} - 7\hat{k}$ ½ m

6. Writing standard form

$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ ½ m

Finding $\theta = \frac{\pi}{2}$ ½ m

SECTION - B

7. $$\begin{array}{l} \text{Family A} \Rightarrow \\ \text{Family B} \Rightarrow \end{array} \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{array}{c} \begin{array}{cc} \text{C} & \text{P} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$ 2 m

Writing Matrix Multiplication as $\begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix}$ 1 m

Writing about awareness of balanced diet 1 m

Alt : Method

Taking the given data for all Men, all Women, all Children for each family, the solution must be given marks accordingly

8. $$\tan\left\{\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\} = \tan\left\{\tan^{-1}\left(\frac{2/5}{1-\frac{1}{25}}\right)+\frac{\pi}{4}\right\}$$ 1 m

$$= \tan\left\{\tan^{-1}\left(\frac{5}{12}\right)+\frac{\pi}{4}\right\}$$ 1 m

$$= \frac{\frac{5}{12}+1}{1-\frac{5}{12}} = \frac{17}{7}$$ 1+1 m

9. Writing $C_1 \leftrightarrow C_2$

$$A = -2\begin{vmatrix} 1 & a^3 & a \\ 1 & b^3 & b \\ 1 & c^3 & c \end{vmatrix}$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$A = -2\begin{vmatrix} 0 & a^3-b^3 & a-b \\ 0 & b^3-c^3 & b-c \\ 1 & c^3 & c \end{vmatrix}$$ 1+1 m

$$A = -2(a-b)(b-c)\begin{vmatrix} 0 & a^2+ab+b^2 & 1 \\ 0 & b^2+c^2+bc & 1 \\ 1 & c^3 & c \end{vmatrix}$$ 1 m

$= -2(a-b)\ (b-c)\ \left\{a^2+ab+b^2-b^2-bc-c^2\right\}$ ½ m

$= \ 2\ (a-b)\ (b-c)\ (c-a)\ (a+b+c)$ ½ m

10. $A = IA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1 m

Using elementary row trans formations to get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} A$$ 2 m

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$$ 1 m

OR

$$AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$$ 1 m

$$BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$$ 1 m

$$AC + BC = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ ½ m

$$(A+B)\ C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$ ½ m

$$= \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ 1 m

Yes, $(A+B)\ C = AC + BC$

11. $$f(x) = \begin{cases} -2x+1 & \text{if} \quad x<0 \\ 1 & \text{if} \quad 0 \le x < 1 \\ 2x-1 & \text{if} \quad x \ge 1 \end{cases}$$ 1½ m

Only possible discontinuties are at $x = 0$, $x = 1$

at $x = 0$	:	at $x = 1$
L. H. limit = 1	:	L. H. limit = 1
$f(0)$ = R. H. limit = 1	:	$f(1)$ = R. H. limit = 1

1 m

$\therefore$ f(x) is continuous in the interval $(-1, 2)$ ½ m

At $x = 0$

L. H. D $= -2 \neq$ R. H. D $= 1$ 1 m

$\therefore$ f(x) is not differentiable in the interval $(-1, 2)$

12. $x = a\ (\cos 2t + 2t \sin 2t)$

$y = a\ (\sin 2t - 2t \cos 2t)$

$$\Rightarrow \frac{dx}{dt} = 4\,at\cos 2\,t$$ 1 m

$$\Rightarrow \frac{dy}{dt} = 4\,at\sin\,2\,t$$ 1 m

$$\Rightarrow \frac{dy}{dx} = \tan 2\,t$$ ½ m

$$\Rightarrow \frac{d^2y}{dx^2} = 2\sec^2 2\,t \cdot \frac{dt}{dx}$$ 1 m

$$\frac{d^2y}{dx^2} = \frac{1}{2\,at\cos^3 2\,t}$$ ½ m

13. $\frac{y}{x} = \log x - \log\,(ax+b)$

differentiating w.r.t. x, 1 m

$$= \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{a}{ax+b} = \frac{b}{x\,(ax+b)}$$

$$= x \cdot \frac{dy}{dx} - y = \frac{bx}{(ax+b)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$ 1 m

differentiating w.r.t. x again

$$x\,\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(ax+b)\,b - abx}{(ax+b)^2}$$

$$x\,\frac{d^2y}{dx^2} = \frac{b^2}{(ax+b)^2}$$ 1 m

Writing $\Rightarrow x^3 \dfrac{d^2y}{dx^2} = \left(\dfrac{bx}{ax+b}\right)^2$ .... (2) ½ m

From (1) and (2) $\Rightarrow$

$$x^3 \frac{d^2y}{dx^2} = \left(x \cdot \frac{dy}{dx} - y\right)^2$$ ½ m

14. $I = \displaystyle\int \frac{x + \sin x - x(1+\cos x)}{x(x+\sin x)}\, dx$ 1 m

$= \displaystyle\int \frac{1}{x}\, dx - \int \frac{1+\cos x}{x+\sin x}\, dx$ put $x + \sin x = t$ 2 m

$\Rightarrow (1 + \cos x)\; dx = dt$

$= \log|x| - \log|x+\sin x| + c$ 1 m

OR

$I = \displaystyle\int \frac{(x-1)(x^2+x+1)+1}{(x-1)(x^2+1)}\, dx$ ½ m

$= \displaystyle\int \frac{x^2+x+1}{x^2+1}\, dx + \int \frac{dx}{(x-1)(x^2+1)}$ 1 m

$= \displaystyle\int \left(1 + \frac{x}{x^2+1} + \frac{1}{2}\frac{1}{x-1} - \frac{1}{2}\frac{x}{x^2+1} - \frac{1}{2}\frac{1}{x^2+1}\right) dx$ 1½ m

$= x + \dfrac{1}{4}\log|x^2+1| + \dfrac{1}{2}\log|x-1| - \dfrac{1}{2}\tan^{-1}x + c$ 1 m

15. $I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+4\tan^2 x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{(1+\tan^2 x)(1+4\tan^2 x)} dx$ 1 m

Put $\tan x = t$

$I = \int_0^{\infty} \frac{dt}{(1+t^2)(1+4t^2)} = -\frac{1}{3}\int_0^{\infty} \frac{dt}{1+t^2} + \frac{4}{3}\int_0^{\infty} \frac{dt}{1+(2t)^2}$ 1 m

$= -\frac{1}{3} \tan^{-1} t \Big]_0^{\infty} + \frac{4}{3\times 2} \tan^{-1}(2t)\Big]_0^{\infty}$ 1 m

$= -\frac{1}{3}\left(\frac{\pi}{2}\right) + \frac{2}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{6}$ 1 m

16. $I = -\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{(\sin x - \cos x)^2 - 2^2} dx$ 1½ m

Put $\sin x - \cos x = t \Rightarrow t = -1$ to 0 1 m

$(\cos x + \sin x)\, dx = dt$

$I = -\int_{-1}^{0} \frac{dt}{t^2 - 2^2}$

$= -\frac{1}{4} \log\left|\frac{t-2}{t+2}\right|\Bigg]_{-1}^{0}$ 1 m

$= -\frac{1}{4}\{0 - \log 3\}$

$= \frac{1}{4} \log 3$ ½ m

17. Writing $\vec{d} = \lambda\left(\vec{a} \times \vec{b}\right)$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$ 1 m

$$= \lambda \left(32\,\hat{i} - \hat{j} - 14\,\hat{k}\right) \text{ .... (1)}$$ 1 m

$$\vec{c} \cdot \vec{d} = 27$$

$$\left(2\hat{i} - \hat{j} + 4\,\hat{k}\right) \cdot \lambda \left(32\hat{i} - \hat{j} - 14\,\hat{k}\right) = 27$$

$$9\,\lambda = 27$$ 1 m

$$\lambda = 3$$

$$\therefore \quad \vec{d} = 96\,\hat{i} - 3\,\hat{j} - 42\,\hat{k}$$ 1 m

18. Lines are parallel ½ m

$$\therefore \quad \text{S.D} = \left| \frac{\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b}}{\left|\vec{b}\right|} \right|$$ 1 m

$$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\,\hat{j} + 2\,\hat{k} \text{ and } \rightarrow b = 2\,\hat{i} + 3\,\hat{j} + 4\,\hat{k}$$

$$\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix}, \quad \left|\vec{b}\right| = \sqrt{29}$$ 1½ + ½ m

$\therefore$ S.D $= \left| \frac{2\hat{i} - \hat{k}}{\sqrt{29}} \right| = \frac{\sqrt{5}}{\sqrt{29}}$ or $\frac{\sqrt{145}}{29}$ ½ m

OR

Required equation of plane is

$2x + y - z - 3 + \lambda(5x - 3y + 4z + 9) = 0 \rightarrow (1)$ 1 m

$x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) + 9\lambda - 3 = 0$ 1 m

(1) is parallel to $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

$\therefore\ 2(2 + 5\lambda) + 4(1 - 3\lambda) + 5(-1 + 4\lambda) = 0$

$\Rightarrow\ \lambda = -\frac{1}{6}$ 1 m

(1) $\Rightarrow\ 7x + 9y - 10z - 27 = 0$ 1 m

19. P (step forward) $= \frac{2}{5}$, P (step backword) $= 3/5$ ½ m

He can remain a step away in either of the

ways : 3 steps forward & 2 backwards 1 m

or 2 steps forward & 3 backwards

$\therefore$ required possibility $= {}^5C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^2 + {}^5C_2 \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^3$ 2 m

$= \frac{72}{125}$ ½ m

OR

A die is thrown

Let E_1 be the event of getting 1 or 2

Let E_2 be the event of getting 3, 4, 5 or 6

Let A be the event of getting a tail

$$P(E_1) = 1/3, \; P(E_2) = 2/3$$ 1 m

$$\Rightarrow \; P\left(A/E_1\right) = \frac{3}{8}, \; \& \; P\left(A/E_2\right) = \frac{1}{2}$$ 1 m

$$P\left(E_2/A\right) = \frac{P(E_2) \times P\left(A/E_2\right)}{P(E_1) \times P\left(A/E_1\right) + P(E_2) \times P\left(A/E_2\right)}$$ 1 m

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}}$$

$$= \frac{8}{11}$$ 1 m

SECTION - C

20. Here $R = \{(a, b) : a, b \in \Re \text{ and } a - b + \sqrt{3} \in S$, where

S is the set of all irrational numbers.$\}$

(i) $\forall\, a \in \Re, (a, a) \in R$ as $a - a + \sqrt{3}$ is irrational

$\therefore$ R is reflexive 1½ m

(ii) Let for $a, b \in \Re, (a, b) \in R$ i. e. $a - b + \sqrt{3}$ is irrational

$a - b + \sqrt{3}$ is irrational $\Rightarrow b - a + \sqrt{3} \in S \;\; \therefore (b, a) \in R$

Hence R is symmetric 2 m

(iii) Let $(a, b) \in R$ and $(b, c) \in R$, for $a, b, c \in \Re$

$\therefore \quad a - b + \sqrt{3} \in S$ and $b - c + \sqrt{3} \in S$

adding to get $a - c + 2\sqrt{3} \in S$ Hence $(a, c) \in R$ 2½ m

$\therefore$ R is Transitive

OR

$\forall\, a, b, c, d, e, f \in \Re$

$((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f)$ 1 m

$= (a + c + e, b + d + f) \quad \rightarrow \quad (3)$

$(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f)$ 1 m

$= (a + c + e, b + d + f) \quad \rightarrow \quad (4)$

$\therefore$ * is Associative

Let (x, y) be on identity element in $\Re \times \Re$

$\Rightarrow \quad (a, b) * (x, y) = (a, b) = (x, y) * (a, b)$

$\Rightarrow \quad a + x = a, \; b + y = b$

$x = 0, \; y = 0$ 2 m

$\therefore$ (0, 0) is identity element

Let the inverse element of $(3, -5)$ be $(x_1, y_1,)$

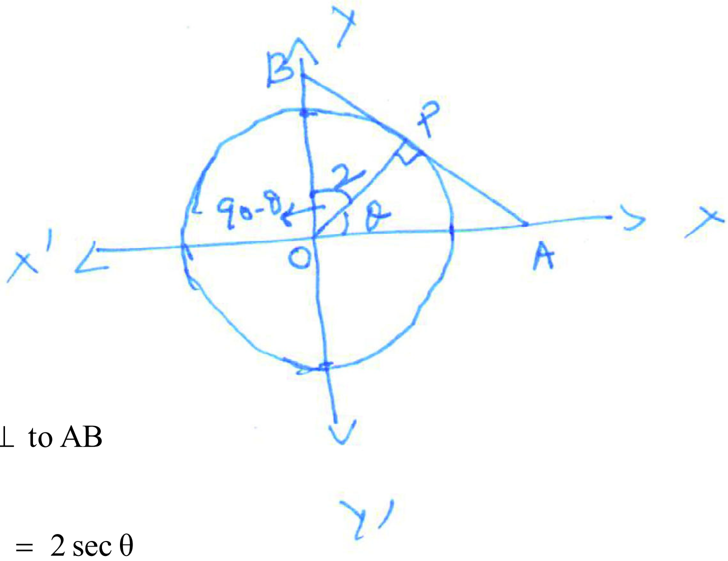
$\Rightarrow \quad (3, -5) * (x_1, y_1) = (0, 0) = (x_1, y_1) * (3, -5)$

$3 + x_1 = 0, \quad -5 + y_1 = 0$

$x_1 = -3, \quad y_1 = 5$

$\Rightarrow$ $(-3, 5)$ is an inverse of $(3, -5)$ 2 m

21. Fig. ½ m



$x^2 + y^2 = 4$. OP is $\perp$ to AB

$\cos\theta = \dfrac{2}{OA}$; $OA = 2\sec\theta$ ½ m

$\cos(90^\circ - \theta) = \dfrac{2}{OB}$

$OB = 2\operatorname{cosec}\theta$ ½ m

Let $S = OA + OB = 2(\sec\theta + \operatorname{cosec}\theta)$ .... (1) 1 m

$\dfrac{dS}{d\theta} = 2(\sec\theta\tan\theta - \operatorname{cosec}\theta \cdot \cot\theta)$ 1m

$= 2\left(\dfrac{\sin^3\theta - \cos^3\theta}{\sin^2\theta \cdot \cos^2\theta}\right)$ .... (2)

for maxima or minima $\dfrac{dS}{d\theta} = 0$

$\Rightarrow \theta = \dfrac{\pi}{4}$, 1 m

(2) $\Rightarrow \dfrac{d^2S}{d\theta^2} > 0$ when $\theta = \dfrac{\pi}{4}$ 1 m

$\therefore$ OA + OB is minimum

$\Rightarrow OA + OB = 4\sqrt{2}$ unit ½ m

22.

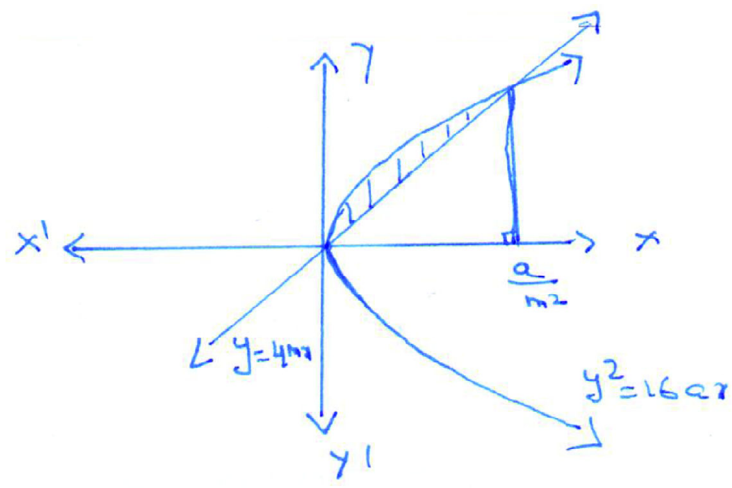


Figure .... ½ m

$$y = 4mx \rightarrow (1) \quad \text{and} \quad y^2 = 16\,ax \rightarrow (2)$$ 1 m

$$\Rightarrow x = \frac{a}{m^2}$$

Required area $= 4\sqrt{a}\int_0^{\frac{a}{m^2}} \sqrt{x}\,dx - 4m\int_0^{\frac{a}{m^2}} x\,dx$ 2 m

$$= \frac{8}{3}\sqrt{a}\,x^{3/2}\Big]_0^{\frac{a}{m^2}} - 2m\,x^2\Big]_0^{\frac{a}{m^2}}$$

$$= \frac{8}{3}\frac{a^2}{m^3} - \frac{2a^2}{m^3} = \frac{2}{3}\frac{a^2}{m^3}$$ 2 m

$$\Rightarrow \frac{2}{3}\cdot a^2/m^3 = \frac{a^2}{12} \text{ given}$$

$$m^3 = 8$$

$$m = 2$$ ½ m

23. $(x-y)\ \frac{dy}{dx} = x+2y$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

$$\frac{dy}{dx} = \frac{1+2\frac{y}{x}}{1-\frac{y}{x}} = f\left({}^{y}\!/\!{}_{x}\right) \text{ .... (1)}$$

$\therefore$ differential equation is homogeneous Eqn. 1 m

$y = vx$ to give

$$v + x\cdot\frac{dv}{dx} = \frac{1+2v}{1-v}$$ ½ m

$$\Rightarrow \int \frac{1-v}{1+v+v^2}\ dv = \int \frac{dx}{x}$$ 1 m

$$\Rightarrow -\frac{1}{2}\int \frac{2v+1}{1+v+v^2}\ dv + \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \int \frac{dx}{x}$$ 1½ m

$$-\frac{1}{2}\log\left|1+v+v^2\right| + \sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = \log|x| + c$$ 1 m

$$-\frac{1}{2}\log\left|\frac{x^2+xy+y^2}{x^2}\right| + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{x\sqrt{3}}\right) = \log|x| + c$$ 1 m

OR

$$(x-h)+(y-k)\ \frac{dy}{dx} = 0$$ 1 m

and $1+(y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ 1 m

$$\Rightarrow \ (y-k) = \frac{-\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$ 1 m

$$(1) \quad \Rightarrow \ (x-h) = \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \ \frac{dy}{dx}$$ 1 m

Putting in the given eqn.

$$\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} \cdot \left(\frac{dy}{dx}\right)^2 + \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$$ 1 m

$$\text{or} \ \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2 \left(\frac{d^2y}{dx^2}\right)^2$$ 1 m

24. Eqn. of a plane through

and Points A (6, 5, 9), B (5, 2, 4) & C (– 1, – 1, 6) is

$$\Rightarrow \begin{vmatrix} x-6 & y-5 & z-9 \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 0$$ 2½ m

$\Rightarrow$ 3x – 4y + 3z – 25 = 0 $\rightarrow$ (2) 1½ m

distance from (3, – 1, 2) to (2)

$$d = \left|\frac{9+4+6-25}{\sqrt{9+16+9}}\right| = \frac{6}{\sqrt{34}} \ \text{units}$$ 2 m

25. Possible values of x are 0, 1, 2 and x is a random variable $1\frac{1}{2}$ m

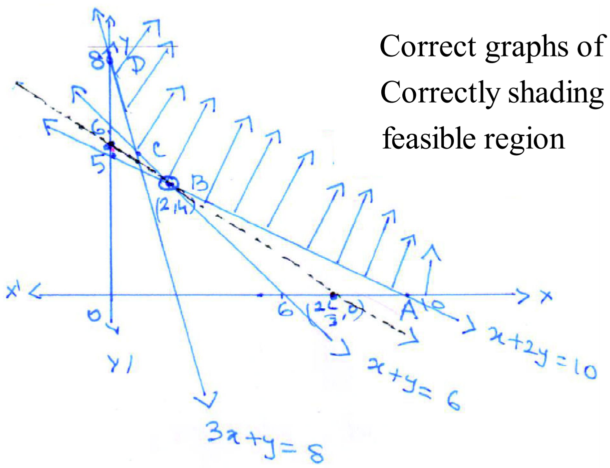
x :	P(x)	x P(x)	x^2 P(x)		
0	$\frac{{}^2C_0 \times {}^5C_2}{{}^7C_2} = \frac{20}{42}$	0	0	For P (x)	$1\frac{1}{2}$ m
1	$\frac{{}^2C_1 \times {}^5C_1}{{}^7C_2} = \frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$	For x P (x)	½ m
2	$\frac{{}^2C_2 \times {}^5C_0}{{}^7C_2} = \frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$	For x^2 P (x)	½ m

$$\sum x\ P(x) = \frac{24}{42};\ \sum x^2\ P(x) = \frac{28}{42}$$ 1 m

$$\text{Mean} = \sum x\ P(x) = \frac{4}{7};\ \text{var}\,iance = \sum x^2\ P(x) - \left[\sum x\ P(x)\right]^2$$ 1 m

$$\text{Variance} = \frac{50}{147} \qquad = \frac{2}{3} - \frac{16}{49} = \frac{50}{147}$$

26.



Correct graphs of 3 lines 3 m
Correctly shading
feasible region ½

Vertices are A (10, 0), B (2, 4), C (1, 5) & D (0, 8) 1 m

Z = 3x + 5y is minimum

at B (2, 4) and the minimum Value is 26. 1 m

on Plotting (3x + 5y < 26)

since these it no common point with the feasible

region, Hence, x = 2, y = 4 gives minimum Z ½ m

QUESTION PAPER CODE 65/2/A

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\overrightarrow{OB} = \dfrac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ ½ m

$\overrightarrow{OC} = 2\vec{b} - \vec{a}$ ½ m

2. Vector Perpendicular to $\vec{a}$ and $\vec{b}$ $= \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

[Finding or using] ½ m

Required Vector $= \hat{i} - 11\hat{j} - 7\hat{k}$ ½ m

3. Writing standard form

$\dfrac{x}{3} = \dfrac{y}{2} = \dfrac{z}{-6}$ and $\dfrac{x}{2} = \dfrac{y}{-12} = \dfrac{z}{-3}$ ½ m

Finding $\theta = \dfrac{\pi}{2}$ ½ m

4. getting $|A| = 1$ ½ m

$|A^n| = 1$ ½ m

5. Order 2 or degree = 1 ½ m

sum = 3 ½ m

6. Writing $\int \dfrac{y}{\sqrt{1+y^2}}\, dy = -\int \dfrac{x\, dx}{\sqrt{1+x^2}}$ ½ m

Getting $\sqrt{1+y^2} + \sqrt{1+x^2} = c$ ½ m

SECTION - B

7. $A = IA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1 m

Using elementary row trans formations to get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} A$$ 2 m

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$$ 1 m

OR

$$AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$$ 1 m

$$BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$$ 1 m

$$AC + BC = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ ½ m

$$(A+B)\ C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$ ½ m

$$= \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ 1 m

Yes, $(A + B)\ C = AC + BC$

8. $$f(x) = \begin{cases} -2x + 1 & \text{if} \quad x < 0 \\ 1 & \text{if} \quad 0 \le x < 1 \\ 2x - 1 & \text{if} \quad x \ge 1 \end{cases}$$ 1½ m

Only possible discontinuities are at $x = 0$, $x = 1$

at $x = 0$	:	at $x = 1$
L. H. limit = 1	:	L. H. limit = 1
$f(0)$ = R. H. limit = 1	:	$f(1)$ = R. H. limit = 1

1 m

$\therefore$ $f(x)$ is continuous in the interval $(-1, 2)$ ½ m

At $x = 0$

L. H. D $= -2 \neq$ R. H. D $= 1$ 1 m

$\therefore$ $f(x)$ is not differentiable in the interval $(-1, 2)$

9. $x = a\ (\cos 2t + 2t \sin 2t)$

$y = a\ (\sin 2t - 2t \cos 2t)$

$\Rightarrow \dfrac{dx}{dt} = 4\,at \cos 2\,t$ 1 m

$\Rightarrow \dfrac{dy}{dt} = 4\,at \sin 2\,t$ 1 m

$\Rightarrow \dfrac{dy}{dx} = \tan 2\,t$ ½ m

$$\Rightarrow \frac{d^2y}{dx^2} = 2\sec^2 2t \cdot \frac{dt}{dx}$$ 1 m

$$\frac{d^2y}{dx^2} = \frac{1}{2\,at\cos^3 2t}$$ ½ m

10. $$\frac{y}{x} = \log x - \log\,(ax+b)$$

differentiating w.r.t. x, 1 m

$$= \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{a}{ax+b} = \frac{b}{x\,(ax+b)}$$

$$= x \cdot \frac{dy}{dx} - y = \frac{bx}{(ax+b)} \quad \text{.................. (1)}$$ 1 m

differentiating w.r.t. x again

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(ax+b)\,b - abx}{(ax+b)^2}$$

$$x\frac{d^2y}{dx^2} = \frac{b^2}{(ax+b)^2}$$ 1 m

Writing $\Rightarrow x^3\dfrac{d^2y}{dx^2} = \left(\dfrac{bx}{ax+b}\right)^2$ .... (2) ½ m

From (1) and (2) $\Rightarrow$

$$x^3\frac{d^2y}{dx^2} = \left(x \cdot \frac{dy}{dx} - y\right)^2$$ ½ m

11. $I = \int \frac{x + \sin x - x(1 + \cos x)}{x(x + \sin x)} dx$ 1 m

$= \int \frac{1}{x} dx - \int \frac{1 + \cos x}{x + \sin x} dx$ put $x + \sin x = t$ $\Rightarrow (1 + \cos x)\ dx = dt$ 2 m

$= \log|x| - \log|x + \sin x| + c$ 1 m

OR

$I = \int \frac{(x-1)(x^2 + x + 1) + 1}{(x-1)(x^2+1)} dx$ ½ m

$= \int \frac{x^2 + x + 1}{x^2 + 1} dx + \int \frac{dx}{(x-1)(x^2+1)}$ 1 m

$= \int \left(1 + \frac{x}{x^2+1} + \frac{1}{2}\frac{1}{x-1} - \frac{1}{2}\frac{x}{x^2+1} - \frac{1}{2}\frac{1}{x^2+1}\right) dx$ 1½ m

$= x + \frac{1}{4} \log|x^2 + 1| + \frac{1}{2} \log|x - 1| - \frac{1}{2} \tan^{-1}x + c$ 1 m

12. $\begin{array}{l} \text{Family A} \Rightarrow \\ \text{Family B} \Rightarrow \end{array} \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{array}{cc} C & P \\ \multicolumn{2}{c}{\begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}} \end{array}$ 2 m

Writing Matrix Multiplication as $\begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix}$ 1 m

Writing about awareness of balanced diet 1 m

Alt : Method

Taking the given data for all Men, all Women, all Children for each family, the solution must be given marks accordingly

13. $\tan\left\{\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\} = \tan\left\{\tan^{-1}\left(\frac{2/5}{1-\frac{1}{25}}\right)+\frac{\pi}{4}\right\}$ 1 m

$$= \tan\left\{\tan^{-1}\left(\frac{5}{12}\right)+\frac{\pi}{4}\right\}$$ 1 m

$$= \frac{\frac{5}{12}+1}{1-\frac{5}{12}} = \frac{17}{7}$$ 1+1 m

14. Writing $C_1 \leftrightarrow C_2$

$$A = -2\begin{vmatrix} 1 & a^3 & a \\ 1 & b^3 & b \\ 1 & c^3 & c \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$ & $R_2 \rightarrow R_2 - R_3$

$$A = -2\begin{vmatrix} 0 & a^3-b^3 & a-b \\ 0 & b^3-c^3 & b-c \\ 1 & c^3 & c \end{vmatrix}$$ 1+1 m

$$A = -2(a-b)(b-c)\begin{vmatrix} 0 & a^2+ab+b^2 & 1 \\ 0 & b^2+c^2+bc & 1 \\ 1 & c^3 & c \end{vmatrix}$$ 1 m

$= -2(a-b)(b-c)\left\{a^2+ab+b^2-b^2-bc-c^2\right\}$ ½ m

$= 2(a-b)(b-c)(c-a)(a+b+c)$ ½ m

15. Writing $\vec{d} = \lambda\left(\vec{a} \times \vec{b}\right)$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$ 1 m

$$= \lambda\left(32\,\hat{i} - \hat{j} - 14\,\hat{k}\right) \;\; .... \;\; (1)$$ 1 m

$$\vec{c} \cdot \vec{d} = 27$$

$$\left(2\hat{i} - \hat{j} + 4\hat{k}\right) \cdot \lambda\left(32\hat{i} - \hat{j} - 14\hat{k}\right) = 27$$

$$9\lambda = 27$$ 1 m

$$\lambda = 3$$

$$\therefore \;\; \vec{d} = 96\,\hat{i} - 3\,\hat{j} - 42\,\hat{k}$$ 1 m

16. Lines are parallel ½ m

$$\therefore \;\; S.D = \left| \frac{\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b}}{\left|\vec{b}\right|} \right|$$ 1 m

$$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\,\hat{j} + 2\,\hat{k} \text{ and } \rightarrow b = 2\,\hat{i} + 3\,\hat{j} + 4\,\hat{k}$$

$$\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix}, \quad \left|\vec{b}\right| = \sqrt{29}$$ 1½ + ½ m

$$\therefore \quad S.D = \left| \frac{2\hat{i} - \hat{k}}{\sqrt{29}} \right| = \frac{\sqrt{5}}{\sqrt{29}} \text{ or } \frac{\sqrt{145}}{29}$$ ½ m

OR

Required equation of plane is

$$2x + y - z - 3 + \lambda(5x - 3y + 4z + 9) = 0 \quad \rightarrow \quad (1)$$ 1 m

$$x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) + 9\lambda - 3 = 0$$ 1 m

(1) is parallel to $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

$$\therefore \quad 2(2 + 5\lambda) + 4(1 - 3\lambda) + 5(-1 + 4\lambda) = 0$$

$$\Rightarrow \quad \lambda = -\frac{1}{6}$$ 1 m

$$(1) \Rightarrow 7x + 9y - 10z - 27 = 0$$ 1 m

17. P (step forward) $= \frac{2}{5}$, P (step backword) $= 3/5$ ½ m

He can remain a step away in either of the

ways : 3 steps forward & 2 backwards 1 m

or 2 steps forward & 3 backwards

$$\therefore \text{ required possibility } = {}^5C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^2 + {}^5C_2 \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^3$$ 2 m

$$= \frac{72}{125}$$ ½ m

OR

A die is thrown

Let E_1 be the event of getting 1 or 2

Let E_2 be the event of getting 3, 4, 5 or 6

Let A be the event of getting a tail

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3}$$ 1 m

$$\Rightarrow \quad P\left(\frac{A}{E_1}\right) = \frac{3}{8}, \ \& \ P\left(\frac{A}{E_2}\right) = \frac{1}{2}$$ 1 m

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \times P\left(\frac{A}{E_2}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)}$$ 1 m

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}}$$

$$= \frac{8}{11}$$ 1 m

18. $$I = \int_0^{\frac{\pi}{2}} \frac{dx}{1 + 4\tan^2 x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{(1 + \tan^2 x)(1 + 4\tan^2 x)} dx$$ 1 m

Put $\tan x = t$

$$I = \int_0^{\infty} \frac{dt}{(1+t^2)(1+4t^2)} = -\frac{1}{3} \int_0^{\infty} \frac{dt}{1+t^2} + \frac{4}{3} \int_0^{\infty} \frac{dt}{1+(2t)^2}$$ 1 m

$$= -\frac{1}{3} \tan^{-1} t \Big]_0^{\infty} + \frac{4}{3 \times 2} \tan^{-1}(2t) \Big]_0^{\infty}$$ 1 m

$$= -\frac{1}{3}\left(\frac{\pi}{2}\right) + \frac{2}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{6}$$ 1 m

19. $I = -\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{(\sin x - \cos x)^2 - 2^2}\, dx$ 1½ m

Put $\sin x - \cos x = t \quad \Rightarrow \quad t = -1$ to 0 1 m

$(\cos x + \sin x)\, dx = dt$

$I = -\int_{-1}^{0} \frac{dt}{t^2 - 2^2}$

$= -\frac{1}{4} \log \left| \frac{t-2}{t+2} \right| \Big]_{-1}^{0}$ 1 m

$= -\frac{1}{4} \{0 - \log 3\}$

½ m

$= \frac{1}{4} \log 3$

SECTION - C

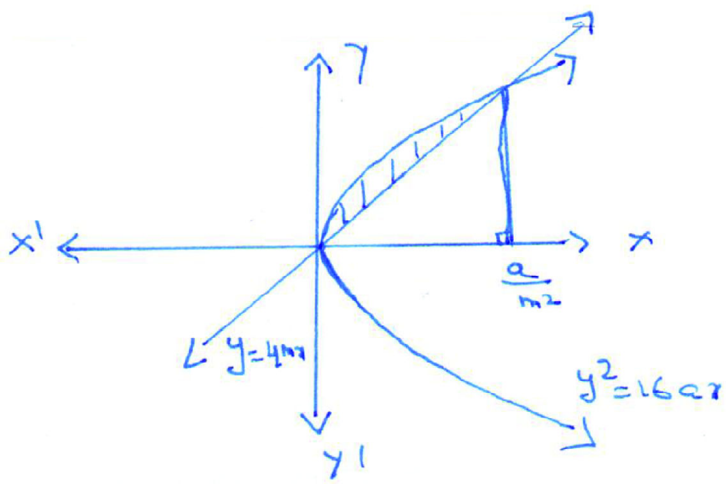
20. 

Figure .... ½ m

$y = 4mx \rightarrow (1)$ and $y^2 = 16\, ax \rightarrow (2)$ 1 m

$\Rightarrow x = \frac{a}{m^2}$

Required area $= 4\sqrt{a}\int_0^{\frac{a}{m^2}} \sqrt{x}\, dx - 4m \int_0^{\frac{a}{m^2}} x\, dx$ 2 m

$$= \frac{8}{3}\sqrt{a}\, x^{3/2}\Big]_0^{\frac{a}{m^2}} - 2m\, x^2\Big]_0^{\frac{a}{m^2}}$$

$$= \frac{8}{3}\frac{a^2}{m^3} - \frac{2a^2}{m^3} = \frac{2}{3}\frac{a^2}{m^3}$$ 2 m

$$\Rightarrow \frac{2}{3} \cdot a^2/m^3 = \frac{a^2}{12} \text{ given}$$

$$m^3 = 8$$

$$m = 2$$ ½ m

21. $(x - y)\dfrac{dy}{dx} = x + 2y$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

$$\frac{dy}{dx} = \frac{1+2\frac{y}{x}}{1-\frac{y}{x}} = f\left(y/x\right) \text{ .... (1)}$$

$\therefore$ differential equation is homogeneous Eqn. 1 m

$y = vx$ to give

$$v + x \cdot \frac{dv}{dx} = \frac{1+2v}{1-v}$$ ½ m

$$\Rightarrow \int \frac{1-v}{1+v+v^2}\, dv = \int \frac{dx}{x}$$ 1 m

$$\Rightarrow -\frac{1}{2}\int \frac{2v+1}{1+v+v^2}\, dv + \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \int \frac{dx}{x}$$ 1½ m

$$-\frac{1}{2}\log\left|1+v+v^2\right| + \sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = \log|x| + c$$ 1 m

$$-\frac{1}{2}\log\left|\frac{x^2+xy+y^2}{x^2}\right| + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{x\sqrt{3}}\right) = \log|x| + c$$ 1 m

OR

$$(x-h)+(y-k)\frac{dy}{dx} = 0$$ 1 m

and $1+(y-k)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 0$ 1 m

$$\Rightarrow (y-k) = \frac{-\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$ 1 m

(1) $\Rightarrow (x-h) = \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\ \frac{dy}{dx}$ 1 m

Putting in the given eqn.

$$\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} \cdot \left(\frac{dy}{dx}\right)^2 + \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$$ 1 m

or $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2 \left(\frac{d^2y}{dx^2}\right)^2$ 1 m

22. Eqn. of a plane through

and Points A (6, 5, 9), B (5, 2, 4) & C (– 1, – 1, 6) is

$$\Rightarrow \begin{vmatrix} x-6 & y-5 & z-9 \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 0$$ 2½ m

$\Rightarrow 3x - 4y + 3z - 25 = 0 \quad \rightarrow \quad (2)$ 1½ m

distance from (3, – 1, 2) to (2)

$$d = \left|\frac{9+4+6-25}{\sqrt{9+16+9}}\right| = \frac{6}{\sqrt{34}} \text{ units}$$ 2 m

23. Here R = {(a, b) : a, b ∈ $\Re$ and $a - b + \sqrt{3} \in S$, where

S is the set of all irrational numbers.}

(i) $\forall\, a \in \Re$, $(a, a) \in R$ as $a - a + \sqrt{3}$ is irrational

$\therefore$ R is reflexive 1½ m

(ii) Let for $a, b \in \Re$, $(a, b) \in R$ i. e. $a - b + \sqrt{3}$ is irrational

$a - b + \sqrt{3}$ is irrational $\Rightarrow b - a + \sqrt{3} \in S \quad \therefore (b, a) \in R$

Hence R is symmetric 2 m

(iii) Let $(a, b) \in R$ and $(b, c) \in R$, for $a, b, c \in \Re$

$\therefore \quad a - b + \sqrt{3} \in S$ and $b - c + \sqrt{3} \in S$

adding to get $a - c + 2\sqrt{3} \in S$ Hence $(a, c) \in R$ 2½ m

$\therefore$ R is Transitive

OR

$\forall$ a, b, c, d, e, f $\in \Re$

$((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f)$ 1 m

$= (a + c + e, b + d + f) \rightarrow (3)$

$(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f)$ 1 m

$= (a + c + e, b + d + f) \rightarrow (4)$

$\therefore$ * is Associative

Let (x, y) be on identity element in $\Re \times \Re$

$\Rightarrow (a, b) * (x, y) = (a, b) = (x, y) * (a, b)$

$\Rightarrow a + x = a, b + y = b$

$x = 0, \; y = 0$ 2 m

$\therefore$ (0, 0) is identity element

Let the inverse element of $(3, -5)$ be $(x_1, y_1,)$

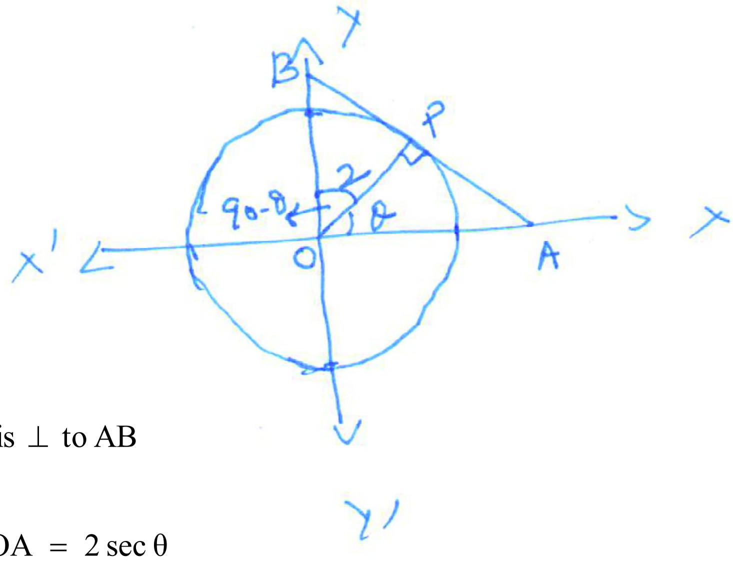
$\Rightarrow (3, -5) * (x_1, y_1) = (0, 0) = (x_1, y_1) * (3, -5)$

$3 + x_1 = 0, \; -5 + y_1 = 0$

$x_1 = -3, \; y_1 = 5$

$\Rightarrow (-3, 5)$ is an inverse of $(3, -5)$ 2 m

24. Fig. ½ m



$x^2 + y^2 = 4$. OP is $\perp$ to AB

$\cos\theta = \dfrac{2}{OA}$; $OA = 2\sec\theta$ ½ m

$\cos(90^\circ - \theta) = \dfrac{2}{OB}$

$OB = 2\,\mathrm{cosec}\,\theta$ ½ m

Let $S = OA + OB = 2(\sec\theta + \mathrm{cosec}\,\theta)$ .... (1) 1 m

$\dfrac{dS}{d\theta} = 2(\sec\theta\tan\theta - \mathrm{cosec}\,\theta\cdot\cot\theta)$ 1m

$= 2\left(\dfrac{\sin^3\theta - \cos^3\theta}{\sin^2\theta\cdot\cos^2\theta}\right)$ .... (2)

for maxima or minima $\dfrac{dS}{d\theta} = 0$

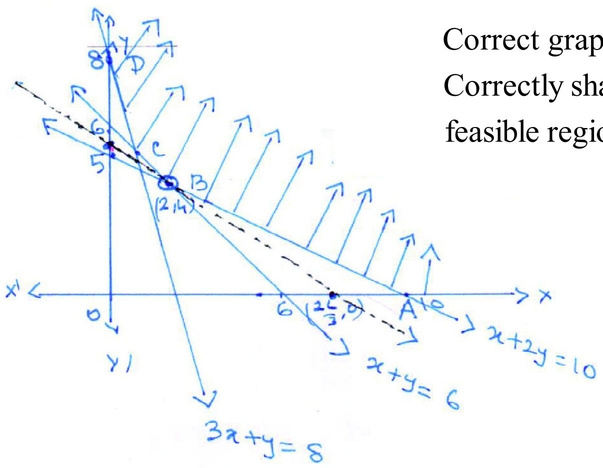
$\Rightarrow \theta = \dfrac{\pi}{4}$, 1 m

(2) $\Rightarrow \dfrac{d^2S}{d\theta^2} > 0$ when $\theta = \dfrac{\pi}{4}$ 1 m

$\therefore$ OA + OB is minimum

$\Rightarrow OA + OB = 4\sqrt{2}$ unit ½ m

25.



Correct graphs of 3 lines 3 m
Correctly shading feasible region ½

Vertices are A (10, 0), B (2, 4), C (1, 5) & D (0, 8) 1 m

Z = 3x + 5y is minimum

at B (2, 4) and the minimum Value is 26. 1 m

on Plotting (3x + 5y < 26)

since these it no common point with the feasible

region, Hence, x = 2, y = 4 gives minimum Z ½ m

26. Possible values of x are 0, 1, 2 and x is a random variable 1½ m

x :	P(x)	x P(x)	x^2 P(x)		
0	$\frac{{}^2C_0 \times {}^5C_2}{{}^7C_2} = \frac{20}{42}$	0	0	For P (x)	1½ m
1	$\frac{{}^2C_1 \times {}^5C_1}{{}^7C_2} = \frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$	For x P (x)	½ m
2	$\frac{{}^2C_2 \times {}^5C_0}{{}^7C_2} = \frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$	For x^2 P (x)	½ m

$$\sum x\ P(x) = \frac{24}{42};\ \sum x^2\ P(x) = \frac{28}{42}$$ 1 m

Mean $= \sum x\ P(x) = \frac{4}{7}$; $\text{var}\,iance = \sum x^2\ P(x) - \left[\sum x\ P(x)\right]^2$ 1 m

Variance $= \frac{50}{147}$ $= \frac{2}{3} - \frac{16}{49} = \frac{50}{147}$

QUESTION PAPER CODE 65/3/A

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. Order 2 or degree = 1 ½ m

   sum = 3 ½ m

2. Writing $\int \frac{y}{\sqrt{1+y^2}}\, dy = -\int \frac{x\, dx}{\sqrt{1+x^2}}$ ½ m

   Getting $\sqrt{1+y^2} + \sqrt{1+x^2} = c$ ½ m

3. getting $|A| = 1$ ½ m

   $|A^n| = 1$ ½ m

4. Vector Perpendicular to $\vec{a}$ and $\vec{b} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

   [Finding or using] ½ m

   Required Vector $= \hat{i} - 11\hat{j} - 7\hat{k}$ ½ m

5. Writing standard form

   $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ ½ m

   Finding $\theta = \frac{\pi}{2}$ ½ m

6. $\overrightarrow{OB} = \frac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ ½ m

   $\overrightarrow{OC} = 2\vec{b} - \vec{a}$ ½ m

SECTION - B

7. $I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+4\tan^2 x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{(1+\tan^2 x)(1+4\tan^2 x)} dx$ 1 m

Put $\tan x = t$

$I = \int_0^{\infty} \frac{dt}{(1+t^2)(1+4t^2)} = -\frac{1}{3}\int_0^{\infty}\frac{dt}{1+t^2} + \frac{4}{3}\int_0^{\infty}\frac{dt}{1+(2t)^2}$ 1 m

$= -\frac{1}{3}\tan^{-1}t\Big]_0^{\infty} + \frac{4}{3\times 2}\tan^{-1}(2t)\Big]_0^{\infty}$ 1 m

$= -\frac{1}{3}\left(\frac{\pi}{2}\right) + \frac{2}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{6}$ 1 m

8. $I = -\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{(\sin x - \cos x)^2 - 2^2} dx$ 1½ m

Put $\sin x - \cos x = t \Rightarrow t = -1$ to 0 1 m

$(\cos x + \sin x)\, dx = dt$

$I = -\int_{-1}^{0} \frac{dt}{t^2 - 2^2}$

$= -\frac{1}{4}\log\left|\frac{t-2}{t+2}\right|\Bigg]_{-1}^{0}$ 1 m

$= -\frac{1}{4}\{0 - \log 3\}$

½ m

$= \frac{1}{4}\log 3$

9. Writing $\vec{d} = \lambda\left(\vec{a} \times \vec{b}\right)$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$ 1 m

$$= \lambda\left(32\hat{i} - \hat{j} - 14\hat{k}\right) \text{ .... (1)}$$ 1 m

$$\vec{c} \cdot \vec{d} = 27$$

$$\left(2\hat{i} - \hat{j} + 4\hat{k}\right) \cdot \lambda\left(32\hat{i} - \hat{j} - 14\hat{k}\right) = 27$$

$$9\lambda = 27$$ 1 m

$$\lambda = 3$$

$$\therefore \quad \vec{d} = 96\hat{i} - 3\hat{j} - 42\hat{k}$$ 1 m

10. Lines are parallel ½ m

$$\therefore \quad S.D = \left| \frac{\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b}}{\left|\vec{b}\right|} \right|$$ 1 m

$$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \rightarrow b = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix}, \quad \left|\vec{b}\right| = \sqrt{29}$$ 1½ + ½ m

$$\therefore \quad S.D = \left| \frac{2\hat{i} - \hat{k}}{\sqrt{29}} \right| = \frac{\sqrt{5}}{\sqrt{29}} \text{ or } \frac{\sqrt{145}}{29}$$ ½ m

OR

Required equation of plane is

$2x + y - z - 3 + \lambda(5x - 3y + 4z + 9) = 0 \quad \to \quad (1)$ 1 m

$x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) + 9\lambda - 3 = 0$ 1 m

(1) is parallel to $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

$\therefore \quad 2(2 + 5\lambda) + 4(1 - 3\lambda) + 5(-1 + 4\lambda) = 0$

$\Rightarrow \quad \lambda = -\frac{1}{6}$ 1 m

(1) $\Rightarrow \quad 7x + 9y - 10z - 27 = 0$ 1 m

11. P (step forward) $= \frac{2}{5}$, P (step backword) $= 3/5$ ½ m

He can remain a step away in either of the

ways : 3 steps forward & 2 backwards 1 m

or 2 steps forward & 3 backwards

$\therefore$ required possibility $= {}^5C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^2 + {}^5C_2 \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^3$ 2 m

$= \frac{72}{125}$ ½ m

OR

A die is thrown

Let E_1 be the event of getting 1 or 2

Let E_2 be the event of getting 3, 4, 5 or 6

Let A be the event of getting a tail

$$P(E_1) = 1/3,\ P(E_2) = 2/3 \qquad 1\text{ m}$$

$$\Rightarrow\ P\left(A/E_1\right) = \frac{3}{8},\ \&\ P\left(A/E_2\right) = \frac{1}{2} \qquad 1\text{ m}$$

$$P\left(E_2/A\right) = \frac{P(E_2)\times P\left(A/E_2\right)}{P(E_1)\times P\left(A/E_1\right) + P(E_2)\times P\left(A/E_2\right)} \qquad 1\text{ m}$$

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}}$$

$$= \frac{8}{11} \qquad 1\text{ m}$$

12. $A = IA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad 1\text{ m}$$

Using elementary row trans formations to get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} A \qquad 2\text{ m}$$

$$\Rightarrow\ A^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} \qquad 1\text{ m}$$

OR

$$AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$$ 1 m

$$BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$$ 1 m

$$AC + BC = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ ½ m

$$(A+B)\ C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$ ½ m

$$= \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ 1 m

Yes, $(A+B)\ C = AC + BC$

13. $$f(x) = \begin{cases} -2x+1 & \text{if} \quad x < 0 \\ 1 & \text{if} \quad 0 \le x < 1 \\ 2x-1 & \text{if} \quad x \ge 1 \end{cases}$$ 1½ m

Only possible discontinuties are at $x = 0,\ x = 1$

at $x = 0$	:	at $x = 1$
L. H. limit = 1	:	L. H. limit = 1
$f(0)$ = R. H. limit = 1	:	$f(1)$ = R. H. limit = 1

1 m

$\therefore\ f(x)$ is continuous in the interval $(-1, 2)$ ½ m

At $x = 0$

L. H. D $= -2 \neq$ R. H. D $= 1$ 1 m

$\therefore\ f(x)$ is not differentiable in the interval $(-1, 2)$

14. $x = a\ (\cos 2t + 2t \sin 2t)$

$y = a\ (\sin 2t - 2t \cos 2t)$

$\Rightarrow \frac{dx}{dt} = 4\ at \cos 2\ t$ 1 m

$\Rightarrow \frac{dy}{dt} = 4\ at \sin\ 2\ t$ 1 m

$\Rightarrow \frac{dy}{dx} = \tan 2\ t$ ½ m

$\Rightarrow \frac{d^2y}{dx^2} = 2 \sec^2 2\ t \cdot \frac{dt}{dx}$ 1 m

$\frac{d^2y}{dx^2} = \frac{1}{2\ at \cos^3 2\ t}$ ½ m

15. $\frac{y}{x} = \log x - \log\ (ax + b)$

differentiating w.r.t. x, 1 m

$= \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{a}{ax+b} = \frac{b}{x\ (ax+b)}$

$= x \cdot \frac{dy}{dx} - y = \frac{bx}{(ax+b)}$ .... (1) 1 m

differentiating w.r.t. x again

$x\ \frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(ax+b)\,b - abx}{(ax+b)^2}$

$$x \frac{d^2y}{dx^2} = \frac{b^2}{(ax+b)^2}$$ 1 m

Writing $\Rightarrow x^3 \frac{d^2y}{dx^2} = \left(\frac{bx}{ax+b}\right)^2$ .... (2) ½ m

From (1) and (2) $\Rightarrow$

$$x^3 \frac{d^2y}{dx^2} = \left(x \cdot \frac{dy}{dx} - y\right)^2$$ ½ m

16. $I = \int \frac{x + \sin x - x(1+\cos x)}{x(x+\sin x)} dx$ 1 m

$= \int \frac{1}{x} dx - \int \frac{1+\cos x}{x+\sin x} dx$ put $x + \sin x = t$ 2 m

$\Rightarrow (1 + \cos x)\ dx = dt$

$= \log|x| - \log|x + \sin x| + c$ 1 m

OR

$I = \int \frac{(x-1)(x^2+x+1)+1}{(x-1)(x^2+1)} dx$ ½ m

$= \int \frac{x^2+x+1}{x^2+1} dx + \int \frac{dx}{(x-1)(x^2+1)}$ 1 m

$= \int \left(1 + \frac{x}{x^2+1} + \frac{1}{2}\frac{1}{x-1} - \frac{1}{2}\frac{x}{x^2+1} - \frac{1}{2}\frac{1}{x^2+1}\right) dx$ 1½ m

$= x + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\log|x-1| - \frac{1}{2}\tan^{-1}x + c$ 1 m

$$\begin{array}{ll} & \begin{array}{cc} C & P \end{array} \\ \text{17.} \quad \begin{array}{l} \text{Family A} \Rightarrow \\ \text{Family B} \Rightarrow \end{array} \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} & \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$ 2 m

Writing Matrix Multiplication as $\begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix}$ 1 m

Writing about awareness of balanced diet 1 m

Alt : Method

Taking the given data for all Men, all Women, all Children for each family, the solution must be given marks accordingly

18. $\tan\left\{\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\} = \tan\left\{\tan^{-1}\left(\frac{2/5}{1-\frac{1}{25}}\right)+\frac{\pi}{4}\right\}$ 1 m

$= \tan\left\{\tan^{-1}\left(\frac{5}{12}\right)+\frac{\pi}{4}\right\}$ 1 m

$= \frac{\frac{5}{12}+1}{1-\frac{5}{12}} = \frac{17}{7}$ 1+1 m

19. Writing $C_1 \leftrightarrow C_2$

$$A = -2\begin{vmatrix} 1 & a^3 & a \\ 1 & b^3 & b \\ 1 & c^3 & c \end{vmatrix}$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$A = -2 \begin{vmatrix} 0 & a^3-b^3 & a-b \\ 0 & b^3-c^3 & b-c \\ 1 & c^3 & c \end{vmatrix}$$ 1+1 m

$$A = -2\,(a-b)\,(b-c) \begin{vmatrix} 0 & a^2+ab+b^2 & 1 \\ 0 & b^2+c^2+bc & 1 \\ 1 & c^3 & c \end{vmatrix}$$ 1 m

$$= -2\,(a-b)\,(b-c)\left\{a^2+ab+b^2-b^2-bc-c^2\right\}$$ ½ m

$$= 2\,(a-b)\,(b-c)\,(c-a)\,(a+b+c)$$ ½ m

SECTION - C

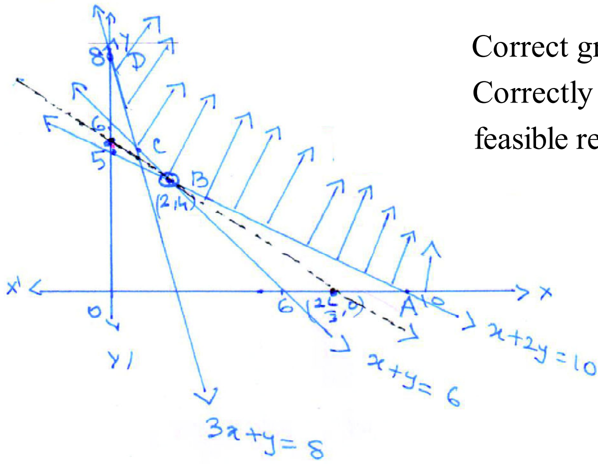
20. Possible values of x are 0, 1, 2 and x is a random variable 1½ m

x :	P(x)	x P(x)	x^2 P(x)		
0	$\frac{{}^2C_0 \times {}^5C_2}{{}^7C_2} = \frac{20}{42}$	0	0	For P (x)	1½ m
1	$\frac{{}^2C_1 \times {}^5C_1}{{}^7C_2} = \frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$	For x P (x)	½ m
2	$\frac{{}^2C_2 \times {}^5C_0}{{}^7C_2} = \frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$	For x^2 P (x)	½ m

$$\sum x\,P(x) = \frac{24}{42};\ \sum x^2\,P(x) = \frac{28}{42}$$ 1 m

$$\text{Mean} = \sum x\,P(x) = \frac{4}{7};\ \text{var}\,iance = \sum x^2\,P(x) - \left[\sum x\,P(x)\right]^2$$ 1 m

$$\text{Variance} = \frac{50}{147} \qquad = \frac{2}{3} - \frac{16}{49} = \frac{50}{147}$$

21. 

Correct graphs of 3 lines 3 m

Correctly shading feasible region ½

Vertices are A (10, 0), B (2, 4), C (1, 5) & D (0, 8) 1 m

$Z = 3x + 5y$ is minimum

at B (2, 4) and the minimum Value is 26. 1 m

on Plotting $(3x + 5y < 26)$

since these it no common point with the feasible

region, Hence, $x = 2,\ y = 4$ gives minimum Z ½ m

22. Here $R = \{(a, b) : a, b \in \Re$ and $a - b + \sqrt{3} \in S$, where

S is the set of all irrational numbers.}

(i) $\forall\ a \in \Re, (a, a) \in R$ as $a - a + \sqrt{3}$ is irrational

$\therefore$ R is reflexive 1½ m

(ii) Let for $a, b \in \Re, (a, b) \in R$ i. e. $a - b + \sqrt{3}$ is irrational

$a - b + \sqrt{3}$ is irrational $\Rightarrow b - a + \sqrt{3} \in S \quad \therefore (b, a) \in R$

Hence R is symmetric 2 m

(iii) Let $(a, b) \in R$ and $(b, c) \in R$, for $a, b, c \in \Re$

$\therefore \quad a - b + \sqrt{3} \in S$ and $b - c + \sqrt{3} \in S$

adding to get $a - c + 2\sqrt{3} \in S$ Hence $(a, c) \in R$ 2½ m

$\therefore$ R is Transitive

OR

$\forall$ a, b, c, d, e, f $\in \Re$

$$((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f)$$

1 m

$$= (a + c + e, b + d + f) \quad \rightarrow \quad (3)$$

$$(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f)$$

1 m

$$= (a + c + e, b + d + f) \quad \rightarrow \quad (4)$$

$\therefore$ * is Associative

Let (x, y) be on identity element in $\Re \times \Re$

$\Rightarrow$ (a, b) * (x, y) = (a, b) = (x, y) * (a, b)

$\Rightarrow$ a + x = a, b + y = b

$x = 0$, $y = 0$ 2 m

$\therefore$ (0, 0) is identity element

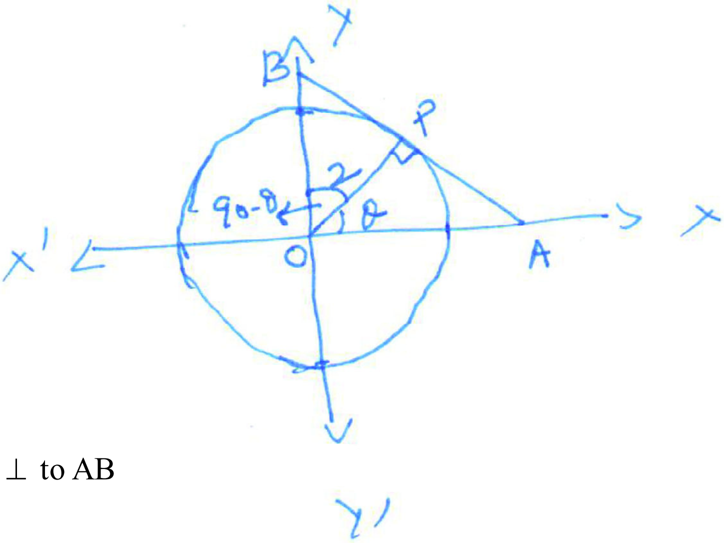
Let the inverse element of (3, – 5) be $(x_1, y_1,)$

$\Rightarrow$ $(3, -5) * (x_1, y_1) = (0, 0) = (x_1, y_1) * (3, -5)$

$3 + x_1 = 0, \quad -5 + y_1 = 0$

$x_1 = -3, \quad y_1 = 5$

$\Rightarrow$ (– 3, 5) is an inverse of (3, – 5) 2 m

23.  Fig. ½ m

$x^2 + y^2 = 4$. OP is $\perp$ to AB

$\cos\theta = \dfrac{2}{OA}$; OA $= 2\sec\theta$ ½ m

$\cos\left(90^\circ - \theta\right) = \dfrac{2}{OB}$

OB $= 2\,\mathrm{cosec}\,\theta$ ½ m

Let S = OA + OB = 2 (sec θ + cosec θ) .... (1) 1 m

$\dfrac{dS}{d\theta} = 2\left(\sec\theta\tan\theta - \mathrm{cosec}\,\theta\cdot\cot\theta\right)$ 1m

$= 2\left(\dfrac{\sin^3\theta - \cos^3\theta}{\sin^2\theta\cdot\cos^2\theta}\right)$ .... (2)

for maxima or minima $\dfrac{dS}{d\theta} = 0$

$\Rightarrow \theta = \dfrac{\pi}{4}$, 1 m

(2) $\Rightarrow \dfrac{d^2S}{d\theta^2} > 0$ when $\theta = \dfrac{\pi}{4}$ 1 m

$\therefore$ OA + OB is minimum

$\Rightarrow$ OA + OB $= 4\sqrt{2}$ unit ½ m

24. $(x-y)\dfrac{dy}{dx} = x+2y$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

$$\frac{dy}{dx} = \frac{1+2\frac{y}{x}}{1-\frac{y}{x}} = f\left(y/x\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$\therefore$ differential equation is homogeneous Eqn. 1 m

$y = vx$ to give

$$v + x\cdot\frac{dv}{dx} = \frac{1+2v}{1-v}$$ ½ m

$$\Rightarrow \int \frac{1-v}{1+v+v^2}\,dv = \int \frac{dx}{x}$$ 1 m

$$\Rightarrow -\frac{1}{2}\int \frac{2v+1}{1+v+v^2}\,dv + \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \int \frac{dx}{x}$$ 1½ m

$$-\frac{1}{2}\log\left|1+v+v^2\right| + \sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = \log|x| + c$$ 1 m

$$-\frac{1}{2}\log\left|\frac{x^2+xy+y^2}{x^2}\right| + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{x\sqrt{3}}\right) = \log|x| + c$$ 1 m

OR

$$(x-h)+(y-k)\frac{dy}{dx} = 0$$ 1 m

and $1+(y-k)\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2 = 0$ 1 m

$$\Rightarrow \quad (y-k) = \frac{-\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$ 1 m

$$(1) \quad \Rightarrow \quad (x-h) = \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \; \frac{dy}{dx}$$ 1 m

Putting in the given eqn.

$$\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} \cdot \left(\frac{dy}{dx}\right)^2 + \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$$ 1 m

$$\text{or} \quad \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2 \left(\frac{d^2y}{dx^2}\right)^2$$ 1 m

25. Eqn. of a plane through

and Points A (6, 5, 9), B (5, 2, 4) & C (– 1, – 1, 6) is

$$\Rightarrow \begin{vmatrix} x-6 & y-5 & z-9 \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 0$$ 2½ m

$\Rightarrow \; 3x - 4y + 3z - 25 = 0 \quad \rightarrow \quad (2)$ 1½ m

distance from (3, – 1, 2) to (2)

$$d = \left| \frac{9+4+6-25}{\sqrt{9+16+9}} \right| = \frac{6}{\sqrt{34}} \text{ units}$$ 2 m

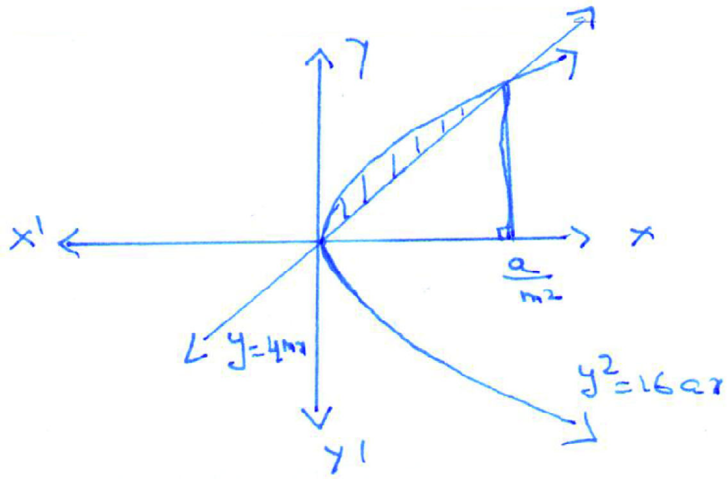
26. 

Figure .... ½ m

$y = 4mx \rightarrow (1)$ and $y^2 = 16\,ax \rightarrow (2)$ 1 m

$$\Rightarrow x = \frac{a}{m^2}$$

Required area $= 4\sqrt{a}\int_0^{\frac{a}{m^2}} \sqrt{x}\,dx - 4m\int_0^{\frac{a}{m^2}} x\,dx$ 2 m

$$= \frac{8}{3}\sqrt{a}\,x^{3/2}\Big]_0^{\frac{a}{m^2}} - 2m\,x^2\Big]_0^{\frac{a}{m^2}}$$

$$= \frac{8}{3}\frac{a^2}{m^3} - \frac{2a^2}{m^3} = \frac{2}{3}\frac{a^2}{m^3}$$ 2 m

$$\Rightarrow \frac{2}{3}\cdot a^2/m^3 = \frac{a^2}{12} \text{ given}$$

$m^3 = 8$

$m = 2$ ½ m