Senior School Certificate Examination

March — 2008

Marking Scheme — Mathematics (Delhi) 65/1/1, 65/1/2, 65/1/3

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

QUESTION PAPER CODE 65/1/1

EXPECTED ANSWERS/VALUE POINTS

SECTION 'A'

1. 7 — 1 m

2. 1 — 1 m

3. $x = 3, \ y = 3$ — 1 m

4. $a^2 + b^2 + c^2 + d^2$ — 1 m

5. 46 — 1 m

6. $\frac{1}{3} \log \left|\left(1+x^3\right)\right| + c$ — 1 m

7. $\frac{\pi}{4}$ — 1 m

8. $\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$ — 1 m

9. $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$ — 1 m

10. $\lambda = \frac{5}{2}$ — 1 m

SECTION 'B'

11. (i) If a candidate writes that the given operation is not a binary operation, give full credit

OR

(ii) If the candidate verifies that the * operation is commutative but not associative, full credit may be given

12. $\text{LHS} = \left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5}\right) + \left(\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}\right)$

$= \tan^{-1}\frac{8/15}{14/15} + \tan^{-1}\frac{15/56}{55/56}$ (1½+1½) m

$= \tan^{-1}\frac{4}{7} + \tan^{-1}\frac{3}{11} = \tan^{-1}\frac{65/77}{65/77} = \tan^{-1}1 = \pi/4 = \text{RHS}$ 1 m

13. $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}, \quad A' = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}$ 1 m

$\therefore \quad \frac{A + A'}{2} = \frac{1}{2}\begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix} = \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix} \Rightarrow$ Symmetric 1 m

$\frac{A - A'}{2} = \frac{1}{2}\begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix} \Rightarrow$ Skew symmetric 1 m

$\therefore \quad \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} = \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix}$ 1 m

OR

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$ 2 m

$$4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}, \quad 5I = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$ 1 m

$$\therefore A^2 - 4A - 5I = \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$ 1 m

14. Getting LHL = 5 1½ m

RHL = 5 1½ m

$\Rightarrow k = 5$ 1 m

15. Let $x = \cos 2\theta$, $\sqrt{1+x} = \sqrt{2}\cos\theta$, $\sqrt{1-x} = \sqrt{2}\sin\theta$ 1 m

Let $y = \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right] = \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right] = \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right)$ 1+½ m

$= \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$ ½ m

$\therefore \frac{dy}{dx} = \frac{-1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{1}{2\sqrt{1-x^2}}$ 1 m

16. $x = \sin 3t \Rightarrow \frac{dx}{dt} = 3\cos 3t,\ (x)_{t=\frac{\pi}{4}} = \frac{1}{\sqrt{2}},\ (y)_{t=\frac{\pi}{4}} = 0$ ½ + ½ m

$y = \cos 2t \Rightarrow \frac{dy}{dt} = -2\sin 2t$ ½ m

$\therefore \frac{dy}{dx} = \frac{-2}{3}\frac{\sin 2t}{\cos 3t}$ 1 m

$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{-2}{3}\frac{\sin \pi/2}{\cos 3\pi/4}$ ½ m

$= \frac{2\sqrt{2}}{3}$

$\therefore$ Equation of tangent is $y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$

$3y = 2\sqrt{2}\ x - 2$ 1 m

or $3y - 2\sqrt{2}\,x + 2 = 0$

17. $I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

$= \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx = \int_0^{\pi}\frac{(\pi - x)\sin x}{1+\cos^2 x}dx$ 1 m

$\Rightarrow 2I = \pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$ 1 m

$I = \frac{\pi}{2}\cdot 2\int_0^{\pi/2}\frac{\sin x\,dx}{1+\cos^2 x} = -\pi\left[\tan^{-1}(\cos x)\right]_0^{\pi/2}$ 1 m

$= -\pi\left[-\frac{\pi}{4}\right] = \frac{\pi^2}{4}$ (½ + ½) m

18. $(x^2 - y^2)dx + 2xy\,dy = 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ ½ m

This is a homogeneous differential equation

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ ½ m

$$v + x\frac{dv}{dx} = \frac{x^2(v^2-1)}{2vx^2} = \frac{v^2-1}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2-1-2v^2}{2v} = -\frac{1+v^2}{2v}$$

$$\Rightarrow \frac{2v}{1+v^2}dv = -\frac{dx}{x}$$ 1 m

$$\log\left|1+v^2\right| = -\log|x| + \log c = \log\frac{c}{x}$$ 1 m

$$1+v^2 = \frac{c}{x} \Rightarrow 1+\frac{y^2}{x^2} = \frac{c}{x}$$

$$\Rightarrow x^2+y^2 = cx$$

when $x = 1, y = 1, \Rightarrow c = 2$ 1 m

$$\therefore \quad x^2+y^2 = 2x$$

OR

$$\frac{dy}{dx} = \frac{2y-x}{2y+x}$$ ½ m

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ ½ m

$$\therefore \quad v + x\frac{dv}{dx} = \frac{x[2v-1]}{x[2v+1]}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2v-1}{2v+1} - v = \frac{2v-1-2v^2-v}{2v+1}$$

$$= -\frac{2v^2-v+1}{2v+1}$$

$$\frac{2v+1}{2v^2-v+1}dv = -\frac{dx}{x}$$ 1 m

$$\frac{1}{2}\frac{4v-1+3}{2v^2-v+1}dv = -\frac{dx}{x}$$

$$\frac{1}{2}\,\frac{4v-1}{2v^2-v+1}\,dv+\frac{3}{4}\,\frac{dv}{v^2-\frac{1}{2}v+\frac{1}{2}}=-\frac{dx}{x}$$ 1 m

$$\frac{1}{2}\log\left|2v^2-v+1\right|+\frac{3}{4}\times\frac{4}{\sqrt{7}}\tan^{-1}\frac{v-\frac{1}{4}}{\frac{\sqrt{7}}{4}}=-\log x+c$$

$$\frac{1}{2}\log\left|\frac{2y^2-xy+x^2}{x^2}\right|+\frac{3}{\sqrt{7}}\tan^{-1}\frac{4y-x}{\sqrt{7}\,x}=-\log x+c$$ 1 m

when $x=1,\ y=1\ \Rightarrow\ c=\frac{1}{2}\log 2+\frac{3}{\sqrt{7}}\tan^{-1}\frac{3}{\sqrt{7}}$

19. The given differential equation can be written as

$$\frac{dy}{dx}+\sec^2x\;y=\tan x\cdot\sec^2x$$ ½ m

$\text{I.F} = e^{\tan x}$ 1 m

$\therefore$ The solution is

$$y.e^{\tan x}=\int e^{\tan x}\cdot\tan x\cdot\sec^2x\,dx+c$$ ½ m

Let $\tan x = z\ \Rightarrow\ \sec^2 x\,dx = dz$

$$\therefore \int e^{\tan x}\tan x\sec^2x\,dx=\int z\,e^z dz+c$$

$$= z\cdot e^z - e^z + c = e^z\,(z-1)+c$$ 1 m

$$y\,e^{\tan x}=e^{\tan x}\,(\tan x-1)+c$$ 1 m

$$\text{or } y=(\tan x-1)+ce^{-\tan x}$$

20. Let $\hat{c}=x\hat{i}+y\hat{j}+z\hat{k}$

It is given that $\vec{a}\times\vec{c}=\vec{b}$

$$\vec{a} \times \vec{c} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & 2 \end{bmatrix} = \hat{i}(z-y) - \hat{j}(z-x) + \hat{k}(y-x) = 0\hat{i} + \hat{j} - \hat{k}$$

$$\Rightarrow z - y = 0 \Rightarrow y = z, -z + x = 1, x - y = 1$$

$$\Rightarrow z = x - 1, \; y = x - 1$$

(1+1) m

Also, $\vec{a} \cdot \vec{c} = 3 \Rightarrow x + y + z = 3$ ½ m

$$\therefore \; x + x - 1 + x - 1 = 3 \;\Rightarrow\; x = 5/3$$

$$\Rightarrow z = y = \frac{2}{3} \quad \Rightarrow \vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

(1+½) m

OR

$\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{a} + \vec{b} = -\vec{c}$ 1 m

$\Rightarrow \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\,\vec{a} \cdot \vec{b} = \left|\vec{c}\right|^2$ 1 m

$9 + 25 + 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = 49$ 1 m

$$2 \cdot 3 \cdot 5 \cos\theta = 49 - 34 = 15$$

$$\Rightarrow \cos\theta = \frac{1}{2} \;\Rightarrow \theta = \pi/3$$

1 m

21. Here $\vec{a}_1 = +3\hat{i} + 5\hat{j} + 7\hat{k}, \; \vec{b}_1 = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{a}_2 = -\hat{i} - \hat{j} - \hat{k}, \; \vec{b}_2 = 7\hat{i} - 6\hat{j} + \hat{k}$

1 m

Shortest distance (SD) $= \dfrac{\left|\left(\vec{a}_2 - \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$ 1 m

$\vec{a}_2 - \vec{a}_1 = -4\hat{i} - 6\hat{j} - 8\hat{k}$ ½ m

Finding $\vec{b_1} - \vec{b_2} = 4\hat{i} + 6\hat{j} + 8\hat{k}$ 1 m

$$\therefore \text{ SD } = \frac{|-16-36-64|}{\sqrt{116}} = \frac{116}{\sqrt{116}} = \sqrt{116}$$ ½ m

OR

A general point on the line

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda \text{ is}$$

$x = 3\lambda - 2,\ y = 2\lambda - 1,\ z = 2\lambda + 3$ 1 m

Its distance D from $(1, 2, 3) = 3\sqrt{2}$

$\therefore (3\sqrt{2})^2 = (3\lambda - 3)^2 + (2\lambda - 3)^2 + (2\lambda)^2$ 1 m

$$\Rightarrow \lambda = 0 \text{ or } \lambda = \frac{30}{17}$$ 1 m

$\therefore$ The points are $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ or $(-2, -1, 3)$ (½ + ½) m

22. P (a doublet) $= \frac{1}{6} \Rightarrow p = \frac{1}{6},\ q = \frac{5}{6}$

Probability distribution is given by $\left(\frac{1}{6} + \frac{5}{6}\right)^4$ 1 m

Let X be the number of successes and P (X), the corresponding probability, where X takes values from 0 to 4 ½ m

$\therefore$ The distribution is

X	0	1	2	3	4
P(X)	625/1296	500/1296	150/1296	20/1296	1/1296

2½ m

SECTION 'C'

23. LHS : $\Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$

$R_3 \to R_3 + R_1 \Rightarrow \Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \alpha+\beta+\gamma & \alpha+\beta+\gamma & \alpha+\beta+\gamma \end{vmatrix}$ 1½ m

$= (\alpha+\beta+r) \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$ ½ m

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$\Delta = (\alpha+\beta+\gamma) \begin{vmatrix} \alpha-\beta & \beta-\gamma & \gamma \\ \alpha^2-\beta^2 & \beta^2-\gamma^2 & \gamma^2 \\ 0 & 0 & 1 \end{vmatrix}$ 1+1 m

$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma) \begin{vmatrix} 1 & 1 & \gamma \\ \alpha+\beta & \beta+\gamma & \gamma^2 \\ 0 & 0 & 1 \end{vmatrix}$ 1 m

Expanding by last row to get

$\Delta = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)\,(\alpha+\beta+\gamma) = \text{RHS}$ 1 m

24.

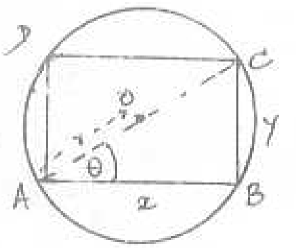


Figure 1 m

Let $\angle$ CAB $= \theta$

$\Rightarrow x = 2r\cos\theta,\ y = 2r\sin\theta$ 1 m

Area $A = x \cdot y = 4r^2 \sin\theta\cos\theta = 2r^2 \sin 2\theta$ 1 m

$\frac{dA}{d\theta} = 4r^2 \cos 2\theta\,;\ \frac{dA}{d\theta} = 0 \Rightarrow \theta = \pi/4$ 1 m

$$\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\pi/4} = \left(-8r^2 \sin 2\theta\right)_{\theta=\pi/4} = -8r^2 < 0$$ 1 m

$\therefore$ A is maximum, at $\theta = \pi/4$

$\Rightarrow$ $x = y$ $\Rightarrow$ Rectangle is a square } 1 m

OR

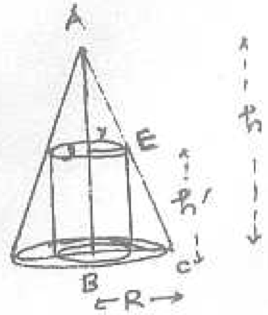


Figure 1 m

$$\Delta ADE \sim \Delta ABC \Rightarrow \frac{h-h'}{h} = \frac{r}{R}$$

$$\Rightarrow r = \frac{R}{h}(h-h')$$

Volume V of cylinder is given by

$$V = \pi r^2 h' = \pi \frac{R^2}{h^2}\left[h^2 + h'^2 - 2hh'\right]h'$$ 2 m

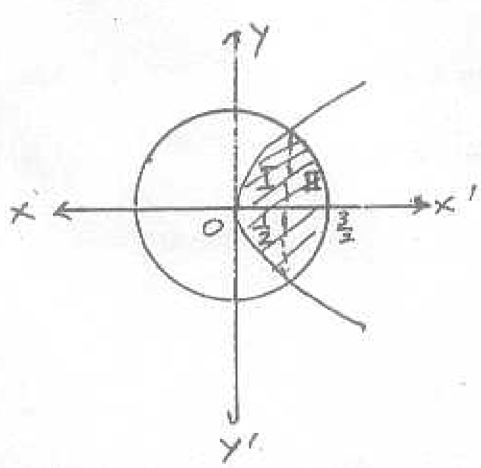
$$\frac{dV}{dh} = \pi \frac{R^2}{h^2}\left[h^2 + 3h'^2 - 4hh'\right]; \frac{dv}{dh} = 0 \Rightarrow h' = \frac{h}{3}$$ 1 m

$$\frac{d^2V}{dh^2} = \frac{\pi R^2}{h^2}\left[-4h + 6h'\right] = \frac{\pi R^2}{h^2}\left[-4h + 2h\right] = -\text{ve}$$ 1 m

$\therefore$ V is maximum at $h' = \frac{1}{3}h$

$\therefore$ Height of cylinder is $\frac{1}{3}h$ 1 m

25. Figure 1 m



Point of intersection, $x = \frac{1}{2}$ 1 m

The required area = I + II

$$= 2\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + 2\int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{\frac{9}{4} - x^2}\,dx$$ 2 m

$$= 2\left|2\cdot\frac{2}{3}x^{3/2}\right| + 2\left|\frac{x\sqrt{\frac{9}{4}-x^2}}{2} + \frac{9}{8}\sin^{-1}\frac{x}{\frac{3}{2}}\right|_{1/2}^{3/2}$$

$$= \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{2} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$ 2 m

[Note : If the candidate finds the other area and gets the

answer as $\frac{9\pi}{8} - \frac{\sqrt{2}}{6} + \frac{9}{4} - \sin^{-1}\frac{1}{3}$, full credit may be given]

26. $I = \int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx = \int_{-a}^{a}\frac{a-x}{\sqrt{a^2-x^2}}\,dx = \underbrace{\int_{-a}^{a}\frac{a\,dx}{\sqrt{a^2-x^2}}}_{I_1} - \underbrace{\int_{-a}^{a}\frac{x\,dx}{\sqrt{a^2-x^2}}}_{I_2}$ (1+1) m

I_1 is even function and I_2 is odd function 1 m

$\therefore\ I_2 = 0$ 1 m

$I = 2a\int_{0}^{a}\frac{dx}{\sqrt{a^2-x^2}} = 2a\cdot\frac{\pi}{2} = \pi a$ 1 m

$\therefore\ I = \pi a$ 1 m

27. The equation of plane passing through (–1, –1, 2) is

a (x+1) + b (y+1) + c (z–2) = 0 where a, b, c

are d.r's of normal to the plane .... (i) 1 m

(i) is $\perp$ to $2x + 3y - 3z = 2$ and $5x - 4y + z = 6$

$\therefore\ 2a + 3b - 3c = 0$

and $5a - 4b + c = 0$ 2 m

$\therefore \quad \frac{a}{-9} = \frac{b}{-17} = \frac{c}{-23}$ 2 m

The d.r's of normal to the plane are 9, 17, 23

$\therefore$ Equation of plane is

$$\left.\begin{aligned} & 9(x+1) + 17(y+1) + 23(z-2) = 0 \\ \Rightarrow\ & 9x + 17y + 23z = 20 \end{aligned}\right\}$$ 1 m

OR

The equation of plane passing through (3, 4, 1) is

$a(x-3) + b(y-4) + c(z-1) = 0$ 1 m

It passes through (0, 1, 0) $\Rightarrow -3a - 3b - c = 0$

$\Rightarrow 3a + 3b + c = 0$ .... (i) 1 m

The plane is parallel to the line $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$

$\Rightarrow$ line is $\perp$ to normal of the plane

$\therefore 2a + 7b + 5c = 0$ .... (ii) 1 m

From (i) and (ii) : $\frac{a}{8} = \frac{b}{-13} = \frac{c}{15}$

$\Rightarrow$ The d.r's are 8, –13, 15 2 m

$\therefore$ Equation of plane is

$$\left.\begin{aligned} & 8(x-3) - 13(y-4) + 15(z-1) = 0 \\ & 8x - 13y + 15z + 13 = 0 \end{aligned}\right\}$$ 1 m

28.

	Machine A	Machine B	Max available
Area needed	1000 m^2	1200 m^2	9000 m^2
Labour Force	12	8	72
Daily Out put	60 units	40 units	

Let x and y be the number of machines A and B respectively

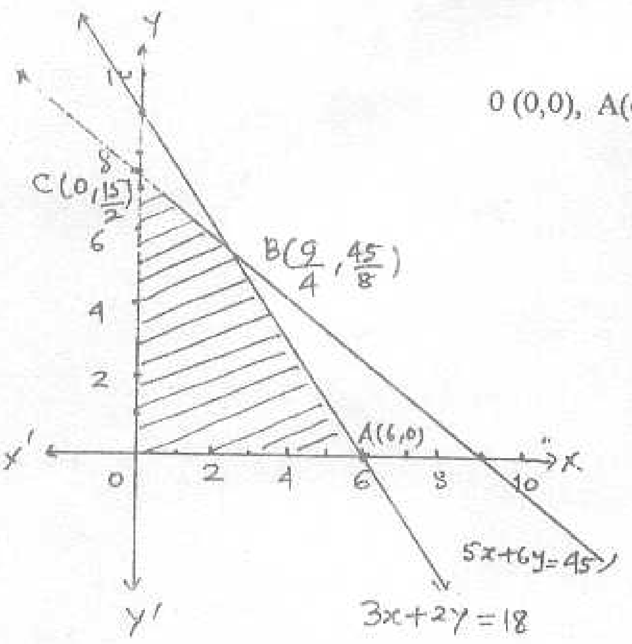
Getting the constraints

$$\left.\begin{array}{l} 1000x + 1200y \le 9000 \Rightarrow 5x + 6y \le 45 \\ 12x + 8y \le 72 \Rightarrow 3x + 2y \le 18 \end{array}\right\}$$ 1½ + ½ m

$$x \ge 0,\ y \ge 0$$

Total output $P = 60x + 40y$ 1 m

The vertices of feasible region are



$0\,(0,0),\ A(6,0),\ B\left(\frac{9}{4},\frac{45}{8}\right),\ c\left(0,\frac{15}{2}\right)$

Correct graph 2 m

$$\left.\begin{array}{l} P(0,0) = 0 \\ P(6,0) = 360 \\ P\left(0,\frac{15}{2}\right) = 300 \\ P\left(\frac{9}{4},\frac{45}{8}\right) = 60 \times \frac{9}{4} + \frac{45}{8} \times 40 \\ \qquad = 135 + 225 = 360 \end{array}\right\}$$ 1 m

$\therefore$ P is equal 360 at A and B

29. Let E_1, E_2, E_3 be the events of a vehicle be a scooter driver, car driver and truck driver.

Let A be the event of a vehicle meeting an accident.

$\therefore\ P(E_1) = \frac{1}{6},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{2}$ (1+½) m

$$P\left(A/E_1\right) = \frac{1}{100}, \; P\left(A/E_2\right) = \frac{3}{100}, \; P\left(A/E_3\right) = \frac{15}{100}$$ (1+½) m

$$P\left(E_1/A\right) = \frac{P(E_1) \times P\left(A/E_1\right)}{\sum_{i=1}^{3} P(E_i) \times \left(A/E_i\right)}, \; i = 1, 2, 3$$ 1 m

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}} = \frac{\frac{1}{6}}{\frac{1}{6} + 1 + \frac{15}{2}} = \frac{1}{6} \times \frac{6}{52} = \frac{1}{52}$$ 2 m

QUESTION PAPER CODE 65/1/3

EXPECTED ANSWERS/VALUE POINTS

SECTION 'A'

1. Same as Q. No. 4 of set 65/1/1.
2. Same as Q. No. 3 of set 65/1/1.
3. Same as Q. No. 1 of set 65/1/1.
4. Same as Q. No. 2 of set 65/1/1.
5. Same as Q. No. 6 of set 65/1/1.
6. Same as Q. No. 5 of set 65/1/1.
7. Same as Q. No. 10 of set 65/1/1.
8. Same as Q. No. 9 of set 65/1/1.
9. Same as Q. No. 8 of set 65/1/1.
10. Same as Q. No. 7 of set 65/1/1.

SECTION 'B'

11. Same as Q. No. 19 of set 65/1/1.
12. Same as Q. No. 18 of set 65/1/1.
13. Same as Q. No. 21 of set 65/1/1.
14. Same as Q. No. 22 of set 65/1/1.
15. Same as Q. No. 20 of set 65/1/1.
16. Same as Q. No. 13 of set 65/1/1.
17. Same as Q. No. 14 of set 65/1/1.

18. Same as Q. No. 16 of set 65/1/1.

19. Same as Q. No. 11 of set 65/1/1.

20. $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$= \tan^{-1}\dfrac{2x+3x}{1-6x^2} = \dfrac{\pi}{4}$ 1 m

$\Rightarrow 6x^2 + 5x - 1 = 0$ 1+½ m

$\Rightarrow (x+1)(6x-1) = 0$

$\Rightarrow x = \dfrac{1}{6}$ and $x = -1$ 1 m

Rejecting $x = -1$, to get $x = \dfrac{1}{6}$ ½ m

21. $I = \int_0^{\pi} \dfrac{x \tan x}{\sec x \operatorname{cosec} x} dx = \int_0^{\pi} x \sin^2 x \, dx$ .... (i) 1 m

$\therefore I = \int_0^{\pi} (\pi - x)\sin^2(\pi - x)\, dx$ .... (ii) 1 m

Adding (i) and (ii) to get

$2I = \pi \int_0^{\pi} \sin^2 x \, dx = \dfrac{\pi}{2}\int_0^{\pi} (1 - \cos 2x)\, dx$ 1 m

$I = \dfrac{\pi}{2}\left[x - \dfrac{\sin 2x}{2}\right]_0^{\pi} = \dfrac{\pi^2}{2}$ 1 m

22. $y = \sqrt{x^2+1} - \log\left[\frac{1}{x} + \frac{\sqrt{x^2+1}}{x}\right] = \sqrt{x^2+1} - \log\left[\frac{1+\sqrt{x^2+1}}{x}\right]$ ½ m

$$\frac{dy}{dx} = \frac{2x}{2\sqrt{x^2+1}} - \left[\frac{x}{1+\sqrt{x^2+1}} \cdot \left\{\frac{\frac{x \cdot x}{\sqrt{x^2+1}} - \left(1+\sqrt{x^2+1}\right) \cdot 1}{x^2}\right\}\right]$$ 1 m

$$= \frac{x}{\sqrt{x^2+1}} - \left[\frac{x}{1+\sqrt{x^2+1}} \cdot \frac{x^2 - \left(1+\sqrt{x^2+1}\right)\sqrt{x^2+1}}{x^2\sqrt{x^2+1}}\right]$$

$$= \frac{x}{\sqrt{x^2+1}} - \left[\frac{x}{1+\sqrt{x^2+1}} \cdot \frac{x^2 - \left(\sqrt{x^2+1} + x^2 + 1\right)}{x^2\sqrt{x^2+1}}\right]$$ 1 m

$$= \frac{x}{\sqrt{x^2+1}} - \left[\frac{x}{1+\sqrt{x^2+1}} \cdot \frac{x^2 - \sqrt{x^2+1} - x^2 - 1}{x^2\sqrt{x^2+1}}\right]$$

$$= \frac{x}{\sqrt{x^2+1}} + \left[\frac{x}{1+\sqrt{x^2+1}} \cdot \frac{1+\sqrt{x^2+1}}{x^2\sqrt{x^2+1}}\right]$$ 1 m

$$= \frac{x}{\sqrt{x^2+1}} + \frac{1}{x\sqrt{x^2+1}} = \frac{x^2+1}{x\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}}{x}$$ ½m

23. $\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$; $R_1 \to R_1 + b\,R_3$ to get 1 m

$$\Delta = \begin{vmatrix} 1+a^2+b^2 & 0 & -b\left(1+a^2+b^2\right) \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)\begin{vmatrix} 1 & 0 & -b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ 1 m

Applying $C_3 \to C_3 + bC_1$ to get $\Delta = (1+a^2+b^2)\begin{vmatrix} 1 & 0 & 0 \\ 2ab & 1-a^2-b^2 & 2a(1+b^2) \\ 2b & -2a & 1-a^2+b^2 \end{vmatrix}$ 2 m

$= (1+a^2+b^2)\left[(1+b^2-a^2)^2 + 4a^2(1+b^2)\right]$

2 m

$= (1+a^2+b^2)\left[(1+a^2+b^2)^2\right] = (1+a^2+b^2)^3$

24. $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$, $\therefore$ $I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx$ 1 m

$I = \int_0^{\pi} \frac{\pi \sin x - x \sin x}{1+\cos^2 x} dx$ 1 m

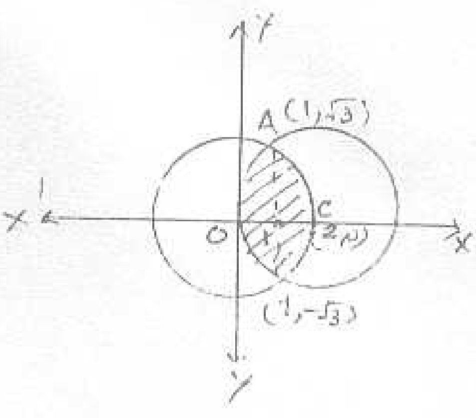
$\therefore$ $2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx = -\pi \int_0^{\pi} \frac{-\sin x}{1+\cos^2 x} dx$ 1+1 m

$= -\pi \left| \tan^{-1}(\cos x) \right|_0^{\pi} = -\pi\left(\frac{-\pi}{4} - \frac{\pi}{4}\right) = \frac{\pi^2}{2}$ 1 m

$\therefore$ $I = \frac{\pi^2}{4}$ 1 m

25. Correct Figure 1 m

Point of Interection : x = 1 1 m



The required area

$= 2$ [area ODAO + area DCAD]

$= 2\left[\int_0^1 \sqrt{4-(x-2)^2}\,dx + \int_1^2 \sqrt{4-x^2}\,dx\right]$ 2 m

$= 2\left[\frac{x-2}{2}\sqrt{4-(x-2)^2} + 2\sin^{-1}\frac{x-2}{2}\right]_0^1 + 2\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]_1^2$

$= -\sqrt{3} + 4\sin^{-1}\left(-\frac{1}{2}\right) - 4\sin^{-1}(-1) + 4\sin^{-1}1 - \sqrt{3} - 4\sin^{-1}\frac{1}{2}$ 1 m

$= -\sqrt{3} - \frac{4\pi}{6} + \frac{4\pi}{2} + \frac{4\pi}{2} - \sqrt{3} - \frac{4\pi}{6}$

$= \left(\frac{8\pi}{3} - -2\sqrt{3}\right)$ sq. units 1m

26. Same as Q. No. 29 of set 65/1/1.

27. Same as Q. No. 27 of set 65/1/1.

28. Same as Q. No. 28 of set 65/1/1.

29. Same as Q. No. 24 of set 65/1/1.

QUESTION PAPER CODE 65/1/3

EXPECTED ANSWERS/VALUE POINTS

SECTION 'A'

1. Same as Q. No. 5 of set 65/1/1.

2. Same as Q. No. 4 of set 65/1/1.

3. Same as Q. No. 1 of set 65/1/1.

4. Same as Q. No. 2 of set 65/1/1.

5. Same as Q. No. 3 of set 65/1/1.

6. Same as Q. No. 7 of set 65/1/1.

7. Same as Q. No. 8 of set 65/1/1.

8. Same as Q. No. 6 of set 65/1/1.

9. Same as Q. No. 10 of set 65/1/1.

10. Same as Q. No. 9 of set 65/1/1.

SECTION 'B'

11. Same as Q. No. 13 of set 65/1/1.

12. Same as Q. No. 14 of set 65/1/1.

13. Same as Q. No. 11 of set 65/1/1.

14. Same as Q. No. 16 of set 65/1/1.

15. Same as Q. No. 18 of set 65/1/1.

16. Same as Q. No. 19 of set 65/1/1.

17. Same as Q. No. 22 of set 65/1/1.

18. Same as Q. No. 21 of set 65/1/1.

19. Same as Q. No. 20 of set 65/1/1.

20. $$\text{L.H.S} = \tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2}$$

$$= \tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x-1}{x-2}\cdot\frac{x+1}{x+2}}\right)$$ 2 m

$$= \tan^{-1}\left(\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right)$$

$$= \tan^{-1}\left(\frac{2x^2-4}{-3}\right) = \pi/4$$ 1 + ½ m

$$\Rightarrow \frac{2x^2-4}{-3} = 1 \Rightarrow 2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$ ½ m

21. If a candidate has attempted this question full credit may be given.

22. $$\int_0^1 \cot^{-1}(1-x+x^2)\,dx = \int_0^1 \tan^{-1}\frac{1}{1-x(1-x)}\,dx$$ 1 m

$$= \int_0^1 \tan^{-1}\left[\frac{x+(1-x)}{1-x(1-x)}\right]dx$$

$$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$$ 1 m

$$= \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1} [1-(1-x)] \, dx$$ ½ m

$$= 2\int_0^1 \tan^{-1} x \, dx = 2\left[\{x \tan^{-1} x\}_0^1 - \frac{1}{2} \int_0^1 \frac{2x}{1+x^2} \, dx \right]$$

$$= 2 \left[\{x \tan^{-1} x\}_0^1 - \frac{1}{2} \{\log(1+x^2)\}_0^1 \right]$$ ½ + ½ m

$$= 2\left[\frac{\pi}{4} - \frac{1}{2} \log 2\right] = \frac{\pi}{2} - \log 2$$ ½ m

SECTION 'C'

23. $$\Delta = \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$ to get $\Delta = \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$ 2 m

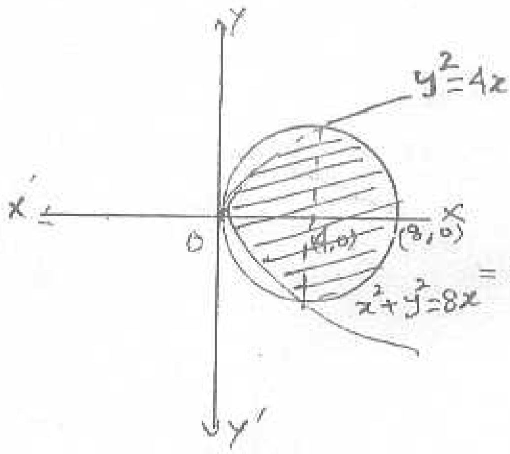
$$= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$ 1 m

Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$ to get

$$\Delta = 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix}$$ 2 m

$$= 2(a+b+c)^3 = \text{RHS}$$ 1 m

24.



Correct Figure 1 m

Point of Interection : $x = 4$ 1 m

Area of shaded region $\int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{16-(x-4)^2}\,dx$

$$= 2\cdot\frac{2}{3}\left[x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + 8\sin^{-1}\frac{x-4}{4}\right]_4^8$$ 2 m

$$= \frac{4}{3}\cdot 8 + 4\pi = \frac{32}{3} + 4\pi$$

$$= \frac{4}{3}(8+3\pi) \text{ sq. units}$$ 2 m

25. $I = \int_0^\pi \frac{x\tan x}{\sec x + \tan x}dx = \int_0^\pi \frac{x\sin x}{1+\sin x}dx$ 1 m

$$I = \int_0^\pi \frac{(\pi-x)\sin(\pi-x)}{1+\sin(\pi-x)}dx = \int_0^\pi \frac{\pi\sin x - x\sin x}{1+\sin x}dx$$ 1 m

$$\therefore\ 2I = \pi\int_0^\pi \frac{\sin x}{1+\sin x}dx = \pi\int_0^\pi\left(1-\frac{1}{1+\sin x}\right)dx$$ 1 m

$$= \pi\int_0^\pi\left(1-\frac{1-\sin x}{\cos^2 x}\right)dx$$ 1 m

$$= \pi\left[(x)_0^\pi - \{\tan x - \sec x\}_0^\pi\right]$$ 1 m

$$= \pi\times\pi - \pi[0+2] = \pi^2 - 2\pi = \pi(\pi-2)$$

$$\therefore\ I = \frac{\pi}{2}(\pi-2)$$ 1 m

26. Same as Q. No. 24 of set 65/1/1.

27. Same as Q. No. 28 of set 65/1/1.

28. Same as Q. No. 29 of set 65/1/1.

29. Same as Q. No. 27 of set 65/1/1.