CHAPTER-4

Linear Equations in Two Variables

Task-1: Sentence to an equation

Topic	Linear equations in two variables
Nature of task	Warm up
Content Coverage	• Linear equations in one variable. • Introduction to the equation in two variables
Learning Objectives	• Recall of linear equations in one variable. • Introduction to the equation in two variables through the extension of linear equation in one variable to two variables.
Task	• Sentence to equation • Solve for x • Expressing linear equation in one variable in terms of two variables e.g. $x = 7$ as $1(x) + 0(y) = 7$
Execution of task	This can be a group activity in the classroom. Teacher may divide the students into four to five groups. Each group would be given two sentences, and students would be asked to frame a linear equation.
Duration	1 period
Criteria for assessment	Teacher may ask questions in groups and observe the level of understanding. It is not necessary to give marks for this assessment. It may be used for diagnostic purpose.
Follow up	–

Group Activity Questionnaire:

1. In a one day International Cricket match between India and Sri Lanka, two Indian batsman together scored 185 runs.

2. Solve for x

(1) $5x + 2 = 12$ (2) $2x - 3 = 5$ (3) $3 = 5x - 2$

(4) $5x - 2 = 3x + 10$ (5) $6x - 3 = 7x + 4$ (6) $3(2x + 1) - (x - 4) = 2$

3. Express the given equations as equations in two variables

(1) $x = -9$ (2) $t = 8$ (3) $5y = 3$ (4) $x = 2$

Task-2: Class Worksheet

Topic	Linear equations in two variables
Nature of task	Content
Content Coverage	• coefficients and constants • solutions of linear equations in two variables
Learning Objectives	• To identify coefficients and constants of linear equations in two variables. • To Prove that a linear equation in two variables has infinitely many solutions and justify their being written as ordered pairs of real numbers.
Task	Class Worksheet
Execution of task	A class worksheet can be given to students for diagnosing the understanding of concept.
Duration	1 period
Criteria for assessment	It is not necessary to give marks to this worksheet. It will be a part of C.W. assessment.
Follow up	–

Class Worksheet

Q.1. Fill in the following missing entries.

Linear equation	Write in $ax + by + c = 0$	Coefficient of x	Coefficient of y	Constant term
$2x + 3y = 5$				
$3x - 2y = 7$				
$4x = 9$				
$-3x + 5y = -8$				
$5x + 7y = -9$				
$3y = -7$				
$4x = -9$				

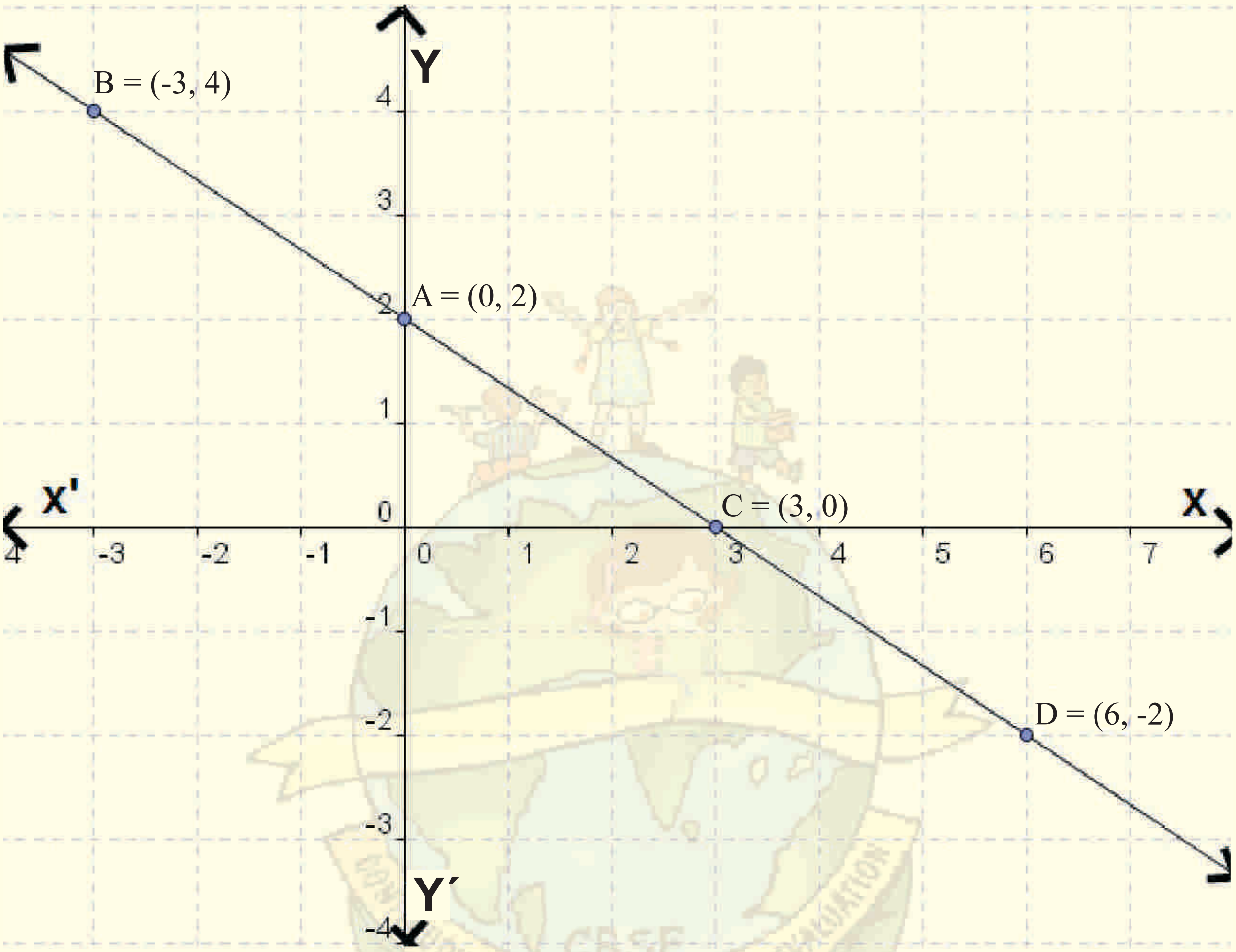
Q.2. Find 5 solutions of the equation $4x + 3y = 12$. How many more solutions are possible ? What do you say?

Task-3: Analysis from graphs

Topic	Linear equations in two variables
Nature of task	Content
Content Coverage	• Linear equations in two variables. • Reading points on a line • Writing equation of line • Lines parallel to coordinate axes
Learning Objectives	• To appreciate that a linear equation in two variables has infinitely many solutions and justify their being written as ordered pairs of real numbers. • To understand the relationship between infinite number of points on a line and infinite solutions of a linear equation in two variables. • To analyze graphs of linear equations
Task	Worksheet containing graphs
Execution of task	This can be an individual or a group activity. Teacher may divide the students into four to five groups. Each group would be given one worksheet, and students would be asked to analyze given graphs.
Duration	2 periods

Worksheet Analysis from graphs:

1. Observe the graph of equation 2x + 3y = 6. Answer the following questions.



Q1. Write the coordinates of points A, B, C, and D.

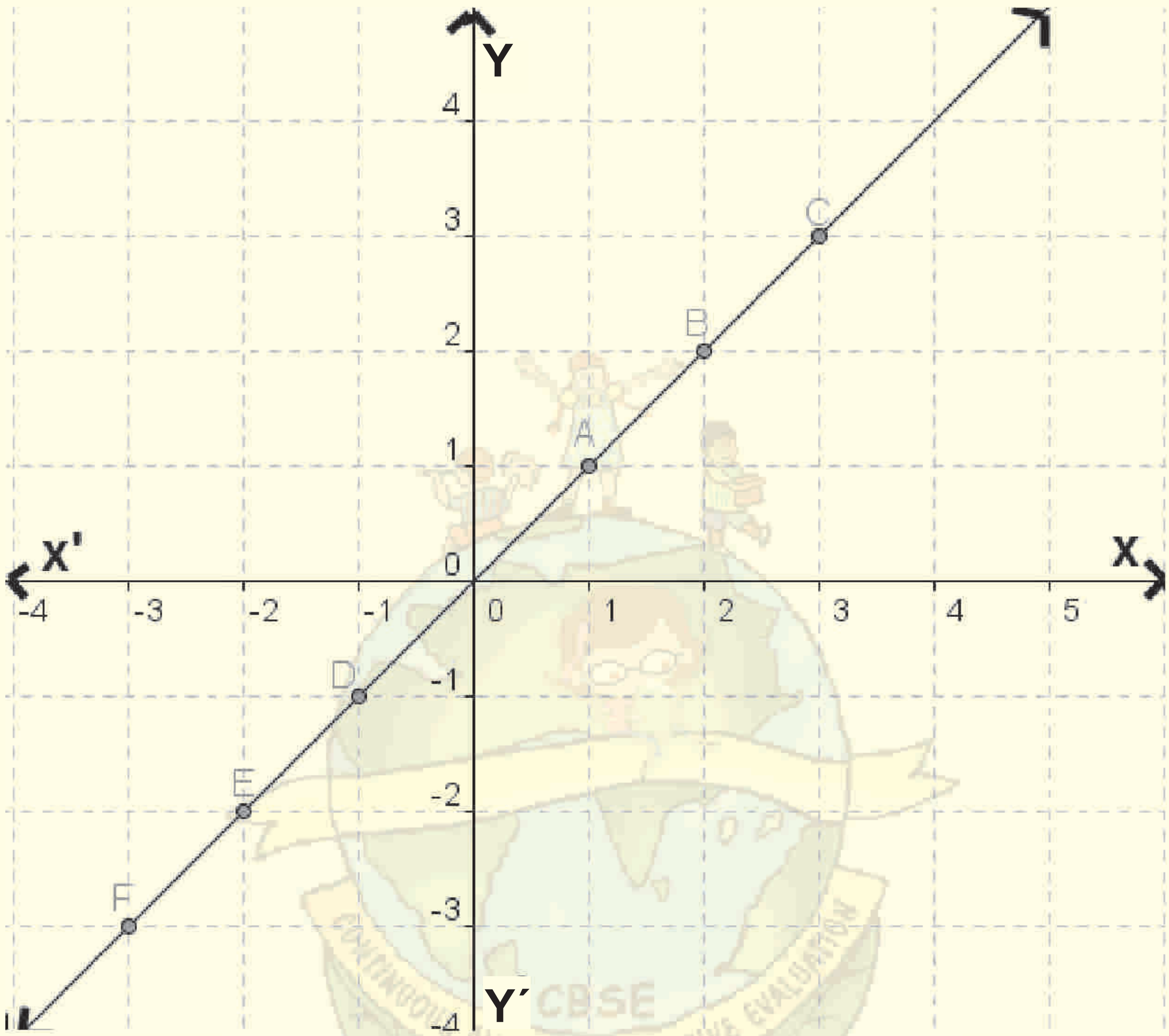
Q2. What is type of graph?

Q3. Can you find two more solutions of this equation?

Q4. How many solutions of the given equation are possible?

Q5. Name the triangle formed by given line and coordinate axes.

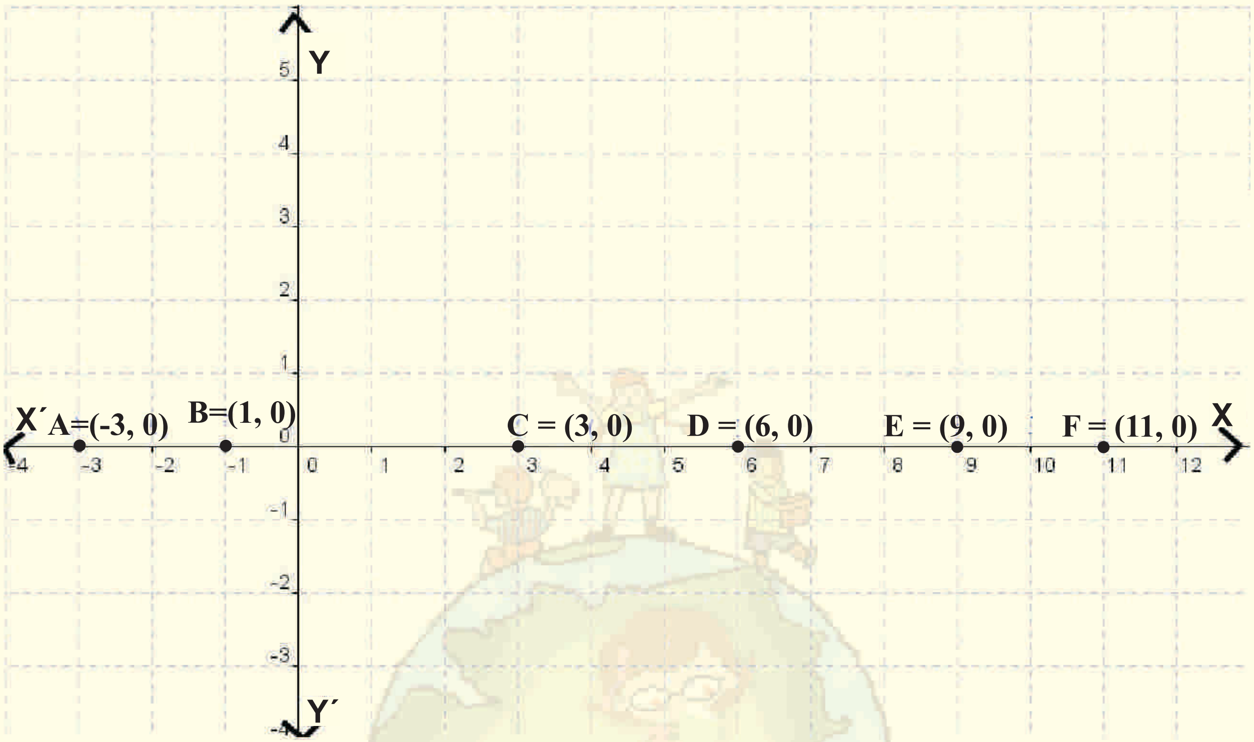
2. **Read the given graph and answer the following questions:**



Q1. Write the coordinates of points A, B, C, D, E, and F.

Q2. What would be the equation of this line.

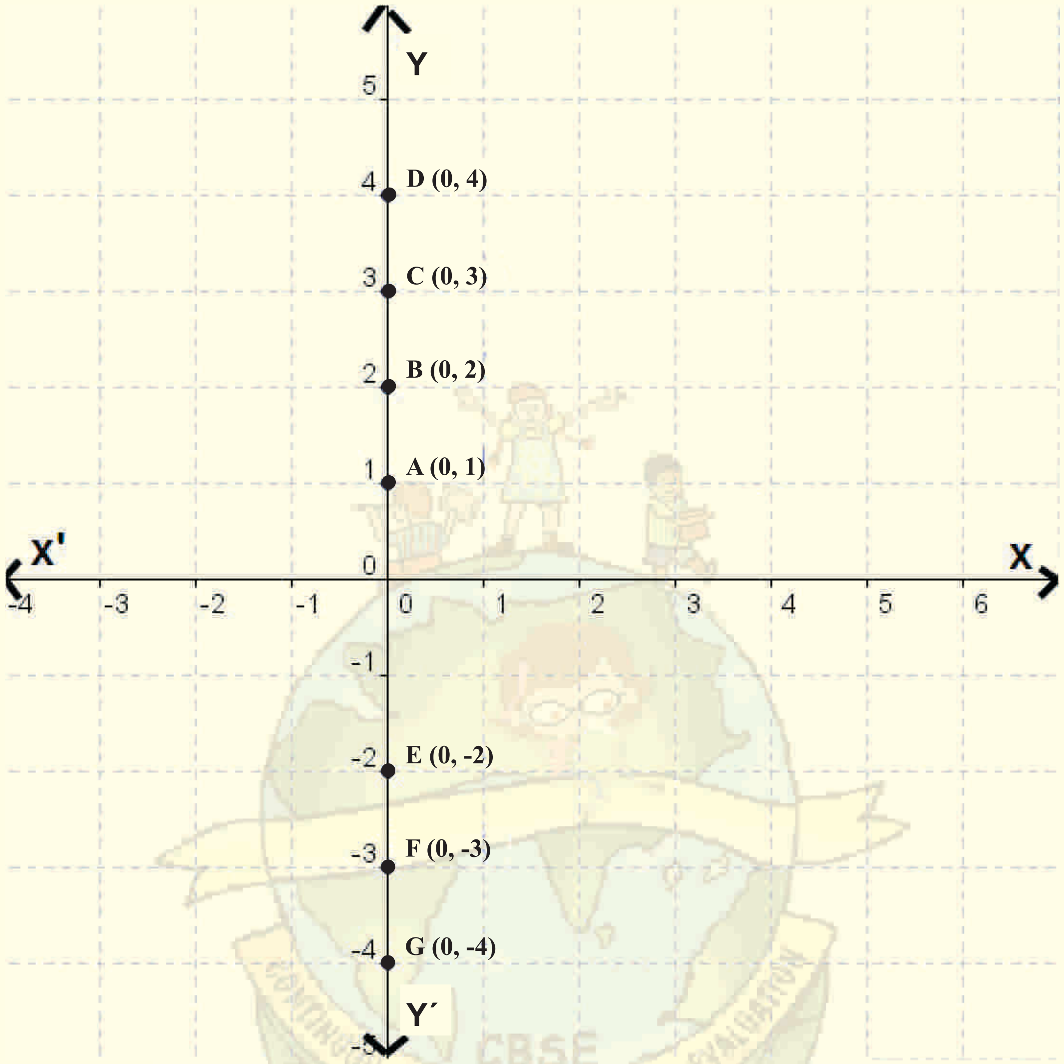
3. Read the given graph and answer the following questions:



Point	Location	Coordinates	Abscissa	Ordinate
A	*x*-axis	(–3, 0)	–3	0
B				
C				
D				
E				
F				

1. What are the coordinates of a general point on the *x*-axis?
2. What is the equation of *x*-axis?

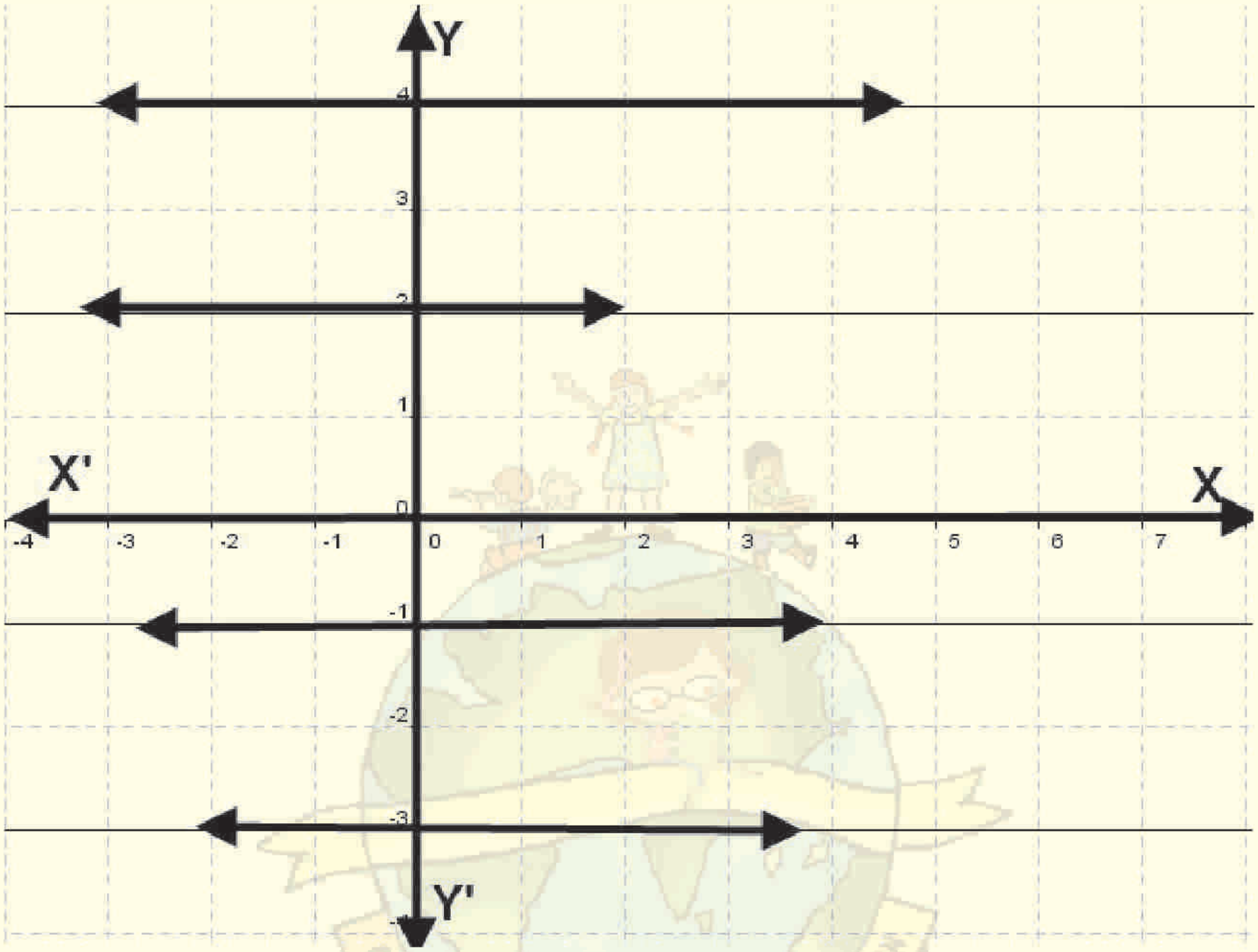
4. Read the given graph and answer the following questions:



Point	Location	Coordinates	Abscissa	Ordinate
A	*y*-axis	(0, 1)	0	1
B				
C				
D				
E				
F				

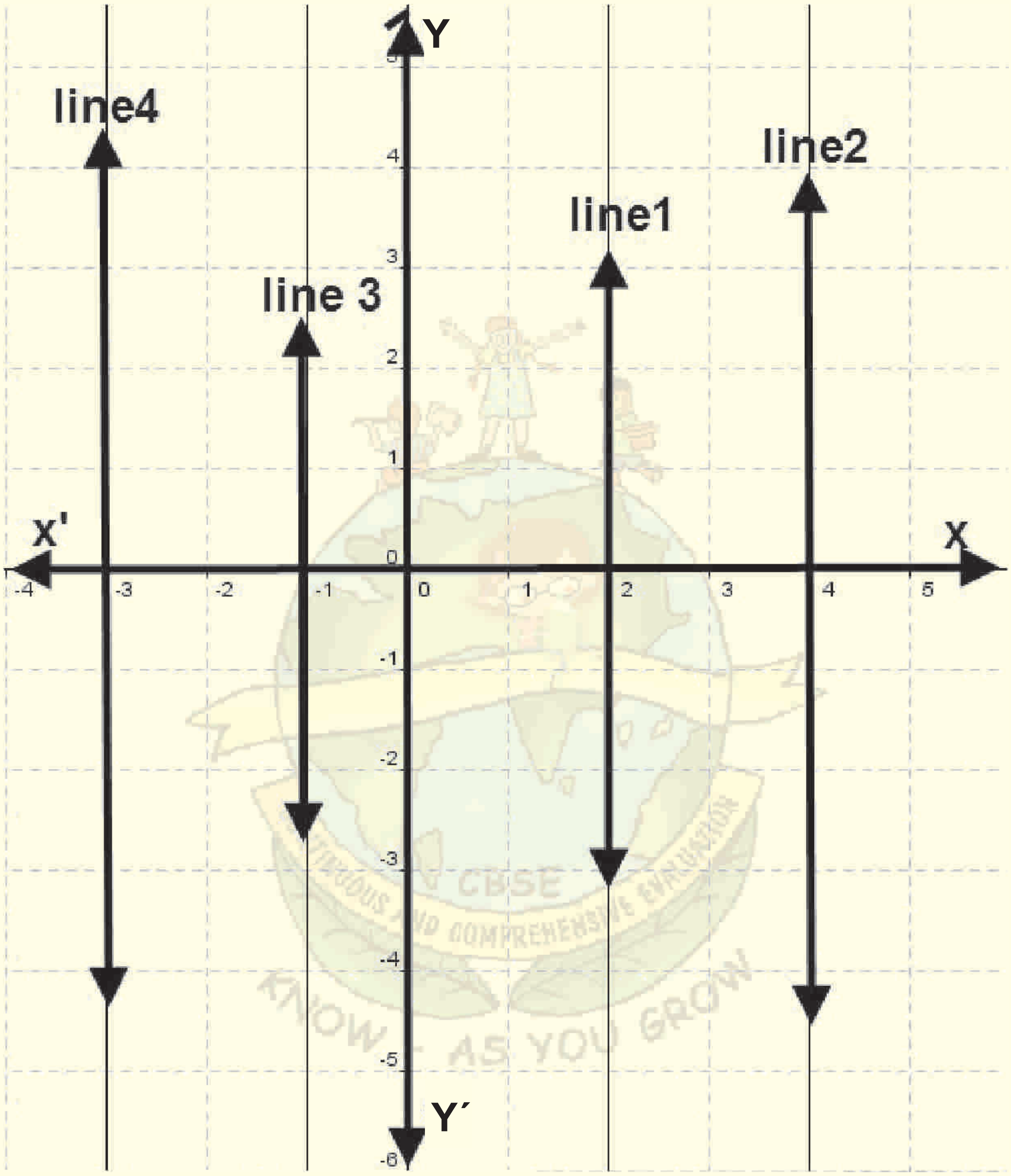
1. What are the coordinates of a general point on the *y*-axis?
2. What is the equation of *y*-axis?

5. Read the given graph and answer the following question:



Q. What do you observe in the given graph? Write at least 5 observations.

6. Read the given graph and answer the following question:



Q. What do you observe in the given graph? Write at least 5 observations.

Task-4: MCQ Worksheet

Topic	Linear equations in two variables
Nature of task	Post content
Content Coverage	Complete Chapter
Learning Objectives	• To recognise a linear equation • To find solution of linear equation in two variables • To recognise the equations parallel to x-axis, y-axis etc.
Execution of task	Teacher may give printed worksheet to the students
Duration	1 period
Criteria for assessment	• For each correct answer, 1 mark may be alloted • In case, MCQ is used as practise worksheet then, it is not necessary to assign marks.
Follow up	• Classroom Discussion: Answers to the questions and common errors may be discussed in the class.

MCQ Worksheet

1. Which of the following is not a linear equation in two variables?

A. $ax + by = c$ B. $ax^2 + bx = c$ C. $2x + 3y = 5$ D. $3x + 2y = 6$

2. The graph of $ax + by + c = 0$ is

A. a straight line parallel to x-axis
B. a straight line parallel to y-axis
C. a general straight line
D. a line in the 2nd and 3rd quadrant

3. The solution of a linear equation in two variables is

A. a number which satisfies the given equation
B. an ordered pair which satisfies the given equation
C. an ordered pair, whose respective values when substituted for x and y in the given equation, satisfies it
D. none of these

4. One of the solutions of the linear equation $3x - 4y + 6 = 0$ is

A. (3, 2) B. (3, -2) C. (2, 3) D. (-2, -3)

5. The ordered pair (m, n) satisfies the equation $ax + by + c = 0$ if

A. $c = 0$
B. $an + bm = 0$
C. $am + bn + c = 0$
D. $am + bn - c = 0$

6. The equation of x-axis is

A. $a = 0$ B. $y = 0$ C. $x = 0$ D. $y = k$

7. From the graph of a line, we can find the coordinates of

A. only one point lying on the line

B. only two points only lying on the line

C. only finite number of points lying on the line

D. only infinite number of points lying on the line

8. A linear equation in two variables has

A. no solution B. only one solution

C. only two solutions D. in finitely many solutions

9. An equation of the form $ax + by + c = 0$ represents a linear equation in two variables, if

A. $a = 0$, $b \neq 0$ B. $a \neq 0$, $b = 0$ C. $a = 0$, $b = 0$ D. $a \neq 0$, $b \neq 0$

10. The graph of the linear equation in two variables $y = mx$ is

A. a line parallel to x-axis B. a line parallel to y-axis

C. a line passing through the origin D. not a straight line

Task-5: Home Assignment

Topic	Linear equations in two variables
Nature of task	Post content
Content Coverage	Complete Chapter
Learning Objectives	• To form linear equations in two variables • To draw the graphs of a linear equation in two variable on graph paper.
Execution of task	For extra practise of content taught, home assignment can be given after the completion of Chapter.
Duration	2 to 3 days
Criteria for Assessment	Follow CW / HW / Assignment Rubric.
Follow up	Class discussion. Answers to the questions may be discussed in class room and individual queries may be answered.

Home Assignment

1. The taxi fares in a city are as follows: Rs 15 for first kilometer and Rs. 8 for every subsequent kilometer. Taking the total distance covered as x and total fare as y, write the above as a linear equation.

2. If (4, 2) is a solution of the equation $4x + 3y - k = 0$ find the value of k.

3. Find the coordinates of the points where the graph of the equation $3x + 4y = 24$ intersects x-axis and y-axis.

4. Draw the graphs of the lines $y = -x$ and $x = y$ on the same graph. From the graph, what do you observe.

5. Draw the graph of the line $2x + 3y = 6$ on the graph paper. Write the sum of the intercepts cut by this line on two axes.

6. Do the points (1, 2), (–1, –16) and (0, –7) lie on the linear equation $y = 9x - 7$. If no, give reasons for the same.

7. The graphs of the equations $2x - y = 6$ and $4x + y = 24$ intersect the x-axis at A and B. Can you find the relationship between OA and OB, where O is the origin.

8. By means of graph verify that the point (1, –1) [*i.e.* $x = 1$ and $y = -1$] is a solution of the equation $3x + 2y - 1 = 0$, $2x + y - 1 = 0$

SAMPLE MATERIAL FOR ASSESSING PROFICIENCY IN LINEAR EQUATIONS IN TWO VARIABLES OVER REAL NUMBERS

Sl.No.	Level/perspective	Technical/discipline specific	Physical relevance	Geometrical relevance	Evaluation tool
1.	Pre requisites / entry	• Variable • Constant • Parameter (relative Constant) • Monomial • Polynomial • Degree of a polynomial • coefficients • Linear polynomial in one variable, two variables over real numbers. • Significance of cover R. • Conditions on coefficients • Zero of a polynomial	• A range of real numbers / physical items. • Specific real values / physical factors. • Conditions determining physical phenomena • Physical examples from real life that lead to these expressions • Physical signifi cance • Various physical situation that lead to these situations	• Points and intervals on number line. • Corresponding geometrical relevance. • Point on a line • Point on a plane • Point on a line seen as point in a plane	Q nos. I to XII The assessment will enable the teacher to assess whether the child is able to discriminate the nuances, the finer distinctions of the concepts.
2.	Process / during Stage I	• Linear equation in one variable over real numbers • Linear equation in two variables over real numbers. • Solution of an equation and comparison with zeros of polynomials.	• Physical examples with comparison in analogous polynomial situations. • Physical meaning of solution.	• Representing single equation as lines in a plane.	XIII-XVI
3.	Stage II	• Simultaneous equation • Methods of solution. • Word problems. 1. assigning variables 2. isolating conditions leading to equations. 3. conversion into equations. 4. identifying suitable method of solution.		• Geometrical meaning of System of equation	

Questions:

I. Identify variables, constants,

1. maximum temperature on any day in the month of March 2009 in Chennai
2. Average rainfall in the month of July 2009 in Delhi.
3. Monthly average rainfall in the year 2009 in Delhi.
4. Any student of class X appearing for AISSE 2010.
5. Age of any student registered in class IX of all affiliated schools of CBSE
6. Chairman, CBSE in the year 2010
7. Chairman, CBSE in the year 1990 to 2010
8. Chairman of any examination board of India at a given point of time.
9. Heights of students of a particular school in class XII.
10. Height of a boy on a given day.
11. Height of a boy from 6 years to 12 years of age.
12. Ratio of the circumference of any circle to its diameter.
13. Radius of any ten concentric circles.
14. Centres of ten concentric circles.
15. Acceleration due to gravity on earth.

II. Using a representative symbol say, x, y etc. describe the above statements. (illustration: x:x is the max, temperature of any day in the month of March 2009 in Chennai).

III. The probable intervals for variation in degree Celsius in I)1

a. 10–15 b. 0–20 c. 20–30 d. 30–45

IV. Which of the variables in the above examples assume numerical values ? Assign probable intervals for variation. Represent them on a number line.

V. Pick out monomials, polynomials, linear polynomials from the following :

VI. Consider the following statements :

A. $\sqrt{xy} + 3x^2 + 4$ is a polynomial and $x^2 y$ is a monomial

B. $\sqrt{xy}$ is a monomial and $3x + y + 5$ is a linear polynomial

C. x^2y is a monomial and $3x + y + 5$ is a linear polynomial

D. $x^{1/3} + y^{1/3} - a^{1/3}$ is not a polynomial but $5x^2y$ is a monomial

Which one of the following is a correct assessment ?

a. A and B are false and C and D are true.

b. Only C is correct; A, B and D are false.

C. Only A is false; B, C and D are true.

D. A and C are false and B and D are true.

VII. Give one word to describe the following :

a. It takes values from a given set of values.

b. It is an algebraic expression where the only operation is multiplication.

c. It is the algebraic sum of monomials.

d. It remains invariant irrespective of time and place.

e. It is the sum of exponents of all variables present in a monomial.

f. It is the degree of the highest degree monomial in a polynomial.

VIII. Reframe question in VII above using "Define the following terms".

IX. How do they differ ?

a. Algebraic sum and sum

b. Absolute constant and relative constant

c. Variable and parameter

d. Monomial and polynomial

X. Construct monomials by assigning suitable variables :

a. The area of a rectangle

b. The volume of a cuboid

c. The area of a square

d. The volume of a cube.

XI. State the conditions under which

a. {x, A}, {y, B} where x and y are variables and A and B sets of values from which the variables take values, represent the same variable

b. polynomial ax + by + c is linear over real numbers

c. A point will lie on x-axis

d. A point will lie on y-axis

e. A value will be a zero of a polynomial

XII. Spell out the mode of transition from geometry to algebra

a. A point on a line is a .... in algebra.

b. A point in a plane is .... in algebra.

c. The area of a square of a varying side is a .... in algebra.

d. The perimeter of a rectangle with varying sides is a .... in algebra.

XIII. Match appropriate implications for statements in A with statements in B

A

a. The geometrical meaning of x-coordinate of a point in a plane

b. A point is at a distance of 5 units from the x-axis

c. Set of all points in a plane with reference to the origin

d. Zero of the polynomial $ax + b$

e. A point fully studied through a pair of coordinates

f. Coordinates of a point

g. X coordinate negative and y coordinate positive

h. A polynomial $ax + by + c$ is a linear polynomial over real numbers

B

i. Algebrically it means that its y-coordinate is 5

j Helps to reduce the dependency on geometry and allows different algebraic operations to provide solutions to geometrical problems.

k. Is its distance from the y-axis

l. The point is in the second quadrant

m. Is as though a single point positions at different distances with reference to an initial position

n. c may be zero but both a and b cannot be zero.

o. An ordered pair of real numbers that represent distances covered from initial position along the two different directions of the x-axis and y-axis simultaneously

p. All points on a line parallel to the y-axis

XIV. Find five ordered pairs of (x, y) satisfying the relation

a. $y = 3x + 5$ b. $2x = -3$ c. $3y = 7$

Plot these points in each case; join them, say which of the following is true and which false according to your observation

a. In each case the points lie on a straight line.

b. In the first case they are on a line but not in respect of second and third cases.

c. Only in the first case there are five pairs and only one in the last two.

d. The first case is a line inclined to the x-axis and y-axis and the remaining two lines parallel to y and x-axes respectively.

XV. Rewrite the three equations in the form $p(x, y) = 0$. Relate the ordered pairs (x, y) to the polynomial $p(x, y)$. Comment on the degree of the polynomial $p(x, y)$. Write the most general form under which these equations can be classified.

XVI. Consider the equation ax + by + c = 0, assign real values to a, b, c in three different cases namely, a = 0, b and $c \neq 0$; a and $c \neq 0$, b = 0; $a \neq 0$, b and c = 0; $b \neq 0$, a and c = 0; a, $b \neq 0$, c = 0; and a, b, $c \neq 0$; In each case find three ordered pairs (x, y) which satisfy the equation, i.e. when these values are substituated for the variables x and y, ax + by + c reduces to zero. In each case plot the three ordered pairs on a graph by suitable choice of x and y axes. Join these points(two at a time) and observe the geometrical figure obtained. Find a fourth ordered pair (x, y) different from the three already determined in each case by assigning an arbitrary value to x. Locate this fourth pair as a point and study it vis-à-vis the existing figures in each case. Conversely take a point on the figure and substitute the co ordinates in the equation. Repeat this atleast three to four times with different vlaues/points. Based on your observations state whether the following statements are correct.

a. In each case the three ordered pairs of (x, y) corresponds to points in a plane that are collinear.

b. When both a and b are non zero points lie on lines that are inclined at an angle greater than 0° to both the axes whereas in the cases when one of a or b is zero the points lie on lines parallel to one of the coordinate axes.

c. In all cases where c is zero the lines pass through the origin.

d. When a and b are both non zero and c=0 is a line through origin and inclined to both axes and when one of a or b is zero along with c the lines represent the axes themselves.

e. When a = 0 the point lie on a line parallel to the x-axis or is the x-axis itself according as $c \neq 0$ or c = 0.

f. When b = 0 the points lie on a line parallel to the y-axis or is the y-axis itself according as $c \neq 0$ or c = 0.

g. Any ordered pair (x, y) satisfying the equation lies on the line determined by the initial three lines in each case.

h. The x and y coordinates of any point on the line determined by the initial three points satisfy the equation.

i. The equation ax + by + c = 0 converts geometrically into a line under all conditions.

XVII. A line in a plane in geometry corresponds to a first degree polynomial in x and y equated to zero of the form ax + by + c = 0.

XVIII. ax + by + c = 0 a, b, c real numbers associated with a line in geometry is called a linear equation in x and y.

Let us test your vocabulary ! Choose the correct answer(s).

a. An ordered pair (x, y) satisfying an equation ax + by + c = 0 is a .... of the equation and a .... of the polynomial on the LHS (zero, solution, degree, answer, point).

b. A solution of the equation ax + by + c = 0 corresponds to .... of a point on a .... in a .... (plane, coordinates, line, zero)

XIX. Put in sequence the steps involved in representing a linear equation in x and y as a line in a plane.

1. Join the points.
2. Let the values chosen be as far as possible to get integral values for y, i.e. x value on substitution gives a constant divisible by coefficient of y.
3. Give three values to x.
4. Plot the ordered pairs as points on a co ordinate plane in reference to a pair of x and y axes.
5. Find the value of y by substituting the value of x in the equation.
6. Tabulate the values of x and y.

XX. Represent the following linear equations geometrically (give three or four equations).

A. At the end of the learning process up to the first phase interspersed with the different type of questioning a teacher enables learning different concepts. To assess whether learning has taken place satisfactorily over different facts identified a comprehensive test may be conducted by including questions for each of the components. The scores in this would indicate the total efforts of diagnosis and remedial and position the child in the ladder of learning once again enabling corrective measures. This can be for a reasonable maximum. An innovative teacher may construct cross work puzzle, different quizzing techniques to make such testing interesting.

Phase II

1. Give One Word

a. Two or more equations that exist simultaneously
b. Two linear equations in x and y over R have atleast one solution
c. Two linear equations in x and y over R have no solution
d. The system represents a pair of parallel lines in the plane
e. This is the geometrical representation of a system of linear equations in x and y over R with unique solution
f. This is the geometrical representation of a system of linear equations in x and y over R with infinite solution.

2. What do they mean geometrically ?

a. A consistent system of linear equations in x and y over R.
b. An inconsistent system of linear equations in x and y over R.
c. A system of linear equations in x and y over R with constant term zero.
d. A system of linear equatiions in x and y over R with coefficient of x zero and nonzero and distinct constant terms.
e. A system of linear equations in x and y over R with coefficient of y zero and nonzero and distinct constant terms.

3. State the conditions under which

a. A system of linear equations in x and y over R is consistent

b. A system of linear equatioins in x and y over R has unique solution.

c. A system of linear equations in x and y over R has no solution.

d. A system of linear equations in x and y over R is in consistent.

e. A pair of lines in a plane represent a consistent system of linear equations in x and y over R

f. A pair of lines in a plane represent an inconsistent system of linear equations in x and y over R.

g. A pair of lines in a plane represent a system of linear equations in x and y over R with unique solution.

4. State the algebraic conditions

a. Two lines in a plane are parallel

b. Two lines in a plane are intersecting

c. Two lines in a plane are coincident

d. A system of linear equations in x and y over R is consistent

e. A system of linear equations in x and y over R has unique solution

f. A system of linear equations in x and y over R has no solution

g. A system of linear equations in x and y over R is in consistent

5. State the geometric conditions

a. Two lines in a plane are parallel

b. Two lines in a plane are intersecting

c. Two lines in a plane are coincident

d. A system of linear equations in x and y over R is consistent

e. A system of linear equations in x and y over R is has unique solution

f. A system of linear equations in x and y over R has no solution

g. A system of linear equations in x and y over R is in consistent

6. a. "Two numbers whose sum is 50 differ by 10. Find the numbers."

i. Assign variables

ii. Form equations by specifying the corresponding part of the problem that has given rise to the equation.

iii. Write a problem involving boat-stream situation for the same equations.

iv. Solve the equations using each one of the three methods.

v. Which of the methods was the simplest in terms of time consumed and complexity of calculations?

Note: (such exercises can be considered for different word problems given in the text.)

b. "A clerk bought 10 pens for distribution among staff and one note book for maintaining account and 10 note books and one pen for his son. For the official purchase he pays

Rs. 95 from office account. For personal purchase he pays a price which is less by Rs. 36. Find the prices of one pen and one note book."

i. Assign variables

ii. Form equations by specifying the corresponding part of the problem that has given rise to the equation.

iii. Write a problem involving a two digit number situation for the same equations.

iv. Solve the equations using each one of the three methods.

v. Which of the methods was the simplest in terms of time consumed and complexity of calculations?

7. Put in sequence the steps involved in solving a word problem;

a. Identify appropriate texts from the problem leading to linear equations.

b. Identify the two unknown variables whose values have to be obtained.

c. Choose appropriate method of solution

d. Assign variables

e. Ascertain concept specific rules

f. Identify the concept around which the problem is woven.

(Note: such steps can be developed in each of the individual physical situations leading to linear equations like two digit numbers, numbers, boat-upstream-downstream, work vs. man power etc.)

Developed by:

C.Gurumurthy
Director (Academic)

Sl. No.	Level/ perspective	Technical/ discipline specific	Physical relevance	Geometrical relevance	Evaluation tool
1	Pre requisites/entry				
2	Process/during Stage I Stage II				